



**Cultural, Social and Technical  
Mathematics  
Secondary IV**

**STUDY GUIDE**





This Study Guide has been developed by teachers and consultants with the aim of helping students prepare for the MELS Uniform Examination in Secondary IV CST Mathematics. The production of this guide was possible through funding by an Anglophone community MELS Success Project.

Please note that this document is a “work in progress” and it will be reviewed every school year. Corrections and suggestions should be sent to your school board consultant.

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# PREPARING FOR THE EXAM AND EXAM TAKING STRATEGIES

## Preparing for the Exam

Preparation is key!

- Pay attention to hints your teacher gives you and take notes.
- Pay regular attention in class and ask for help when needed.
- Go to the tutorial sessions (review).
- Do not leave a topic misunderstood hoping it will not be on the exam. It will very likely be on the exam.
- Budget your time, schedule time to study so that you are well prepared for the test (weeks in advance). Do not wait until the day before!
- Create your own clear and well organized memory aid. This requires planning and time.
- Practice with questions from previous MELS Uniform Exams.
- Complete this booklet.
- Have a good night sleep the night before the exam. Go to bed earlier.
- Have a good breakfast. A healthy meal will give you the mental energy you will need to get through it.

## The Day of the Exam

You will need to bring:

- at least two HB pencils and a good eraser
- a calculator (with or without graphic display) but make sure all data and programs are deleted
- a ruler
- your memory aid
- a watch to better pace yourself

Optional:

- a set square, a compass and a protractor
- additional graph paper





## Exam Taking Strategies

- Keep a positive attitude and try to stay relaxed.
- When you first receive your test, do a quick read of the entire test in order to appropriately pace yourself. Look for what is easy and what will require more effort.
- Do the easiest problems first.
- Don't stay on a problem you are stuck on. Come back to it later.
- Read the entire question at least TWICE.
- Watch out for questions with expressions such as: NOT, STRONGEST TO WEAKEST, INCREASING, DECREASING, etc.
- Ask for clarifications, if needed.
- Write legibly and show all your work when required.
- Look over your test (review). Make sure you've answered everything.
- Do not leave any blanks.

## PREPARING A MEMORY AID

A memory aid consists of one letter size (8 ½" x 11") sheet of paper IN YOUR HANDWRITING. BOTH sides may be used.

It should contain important information required for the exam. It should be NEAT and ORGANIZED. Have a plan and then write the elements on the sheet of paper. You might have to make more than one memory aid before being satisfied with the results. It's worth the time and effort.

Make sure to make your OWN memory aid. It is a good way to study for the exam and you will know where to find the items. You may not use someone else's memory aid (it is considered cheating). And besides, copying someone else's memory aid may not help you at all.

What can it contain?

- Formulas
- Example problems worked out
- The steps used in the problem listed in order
- Reminders of things to look out for in doing a problem
- Any rules used to solve problems
- Definitions
- Tips and hints

How can you organize it?

- The information should be organized by topic (e.g. triangles: congruent, similar...).
- Use lines (or boxes) to separate the different topics (e.g. a section for analytic geometry where you would include distance between two points, midpoint, slope, etc.).
- Use the resources your teacher suggests
- Make sure you include items you tend to forget.
- Use a color code system or a numbering system.

## **FORMAT OF THE UNIFORM EXAM**

The exam will consist of 3 parts:

- A. Multiple Choice Questions (6 questions; 4 marks each)
- B. Short Answer Questions (4 questions; 4 marks each)
- C. Application Questions (6 questions; 10 marks each)

### **Part A. Multiple Choice Questions**

You will read the questions from a Question Booklet and will choose a statement (A, B, C or D) that best represents your answer. You will answer on a machine-scored answer sheet by filling in a circle using an HB pencil. Make sure you fill it in completely.

In this section, you do not need to show work for marks. Always work out the problem entirely and check all the distractors. Do not stop reading when you think you found the right answer. Read everything. You will be given 4 marks or 0 for each question.

Do not leave a blank! Make a choice even if you don't know the answer! You have a 25% probability of getting it right.

### **Part B. Short Answer Questions**

You will read the questions from a Question Booklet and will write a statement in the space provided in your Student Booklet.

In this section, you do not need to show work for marks. However, always work out the problem entirely anyways. You will be given 4 marks or 0 for each question. No part marks are given.

Do not leave a blank! Make an educated guess even if you don't know the answer!

## Part C. Application Questions

You will read the questions from your Student Booklet and answer it in the Student Booklet (same booklet).

For each question, you must show all your work to justify your answer.

Your work must be organized and clearly presented and cannot simply involve listing the calculator applications used to obtain results or information.

You will be given a mark of 0 if you do not show work or if your work does not justify your answer (even if you have the correct answer).

You will be graded using the evaluation criteria for competency 2:

- Cr.1 Formulation of a conjecture suited to the situation, if applicable
- Cr.2 Correct use of appropriate mathematical concepts and processes
- Cr.3 Proper implementation of mathematical reasoning suited to the situation
- Cr.4 Proper organization of the steps in an appropriate procedure
- Cr.5 Correct justification of the steps in an appropriate procedure

The scoring will go as follows. The table on the right is for a conjecture situation. Most problems on the exam are scored according to the table on the left.

Observable indicators corresponding to level						
	A	B	C	D	E	
Cr. 3	40	32	24	16	8	0
Cr. 2	40	32	24	16	8	0
Cr. 4 Cr. 5	20	16	12	8	4	0

Observable indicators corresponding to level						
	A	B	C	D	E	
Cr. 3	40	32	24	16	8	0
Cr. 2	20	16	12	8	4	0
Cr. 4 Cr. 5	20	16	12	8	4	0
Cr.1	20	16	12	8	4	0

As you can see, you can easily obtain marks for showing some work. Write down your process first (the steps) and then show all your work. At the very least, list the concepts you think apply, write out the applicable formulas, etc. Try something!

**Do not leave a blank!**

## SKILL LIST

Can you do the following?

Put a check ✓ in the appropriate box

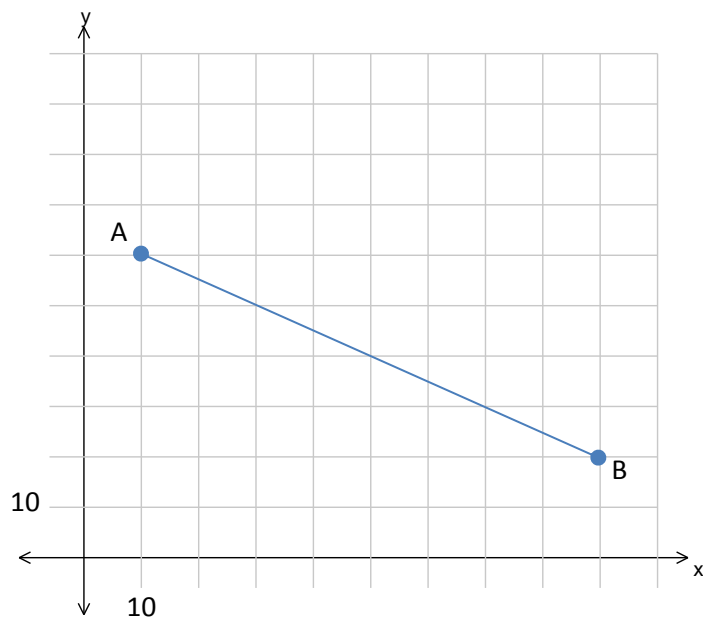
Skill	Yes	Not yet
How to find the <b>DISTANCE</b> between two points		
How to find the <b>MIDPOINT</b> between two points		
How to find the point that divides a line into a given <b>RATIO (part to part)</b>		
How to find the point that divides a line into a given <b>FRACTION (part to whole)</b>		
How to find the <b>slope</b> of a line		
How to express an equation in both <b>STANDARD AND GENERAL FORM</b>		
How to find the <b>equation of a line given the slope and a point on the line</b>		
How to find the <b>equation of a line given two points on the line</b>		
How to find the <b>equation of a line parallel to a given line</b>		
How to find the <b>equation of a line perpendicular to a given line</b>		
How to determine the <b>position of two straight lines</b> (parallel and perpendicular)		
How to translate a story into a <b>SYSTEM OF EQUATIONS</b>		
How to <b>display a system of relations and their solution on a graph</b>		
How to solve a system of equations using the <b>ELIMINATION METHOD</b>		
How to solve a system of equations using the <b>SUBSTITUTION METHOD</b>		
How to solve a system of equations using the <b>COMPARISON METHOD</b>		
How to <b>recognize and translate an INEQUALITY</b>		
How to solve an <b>INEQUALITY graphically</b> and check for feasible region		
<b>How to determine and interpret the following properties in functions:</b>		
• What is a function		
• The domain and range of a function		
• Where the function is increasing, constant and decreasing		
• The minimum and maximum		
• The sign of a function		
• The y-intercept of a function		
• The zeros of a function (x-intercepts)		
<b>How to work with the following functions (words, graph, equation, table):</b>		
• Zero degree		
• First degree (direct)		
• First degree (partial- positive and negative slopes)		
• 2 <sup>nd</sup> degree (quadratic) function $f(x) = ax^2$		
• Exponential function (growth and decay) $f(x) = ac^x$		
• Step function		

• Periodic function		
• Piecewise function		
How to find an <b>angle</b> measure using <b>TRIGONOMETRIC RATIOS (SIN, COS, TAN)</b>		
How to find a <b>side</b> measure using <b>TRIGONOMETRIC RATIOS (SIN, COS, TAN)</b>		
How to find an <b>angle or side</b> measure using <b>SINE LAW</b>		
How to find the <b>AREA OF A TRIANGLE</b> - all three methods:		
• General formula		
• Hero's formula		
• Trigonometric formula		
How to apply <b>CLASSIFICATION OF TRIANGLES</b>		
How to use <b>PYTHAGOREAN THEOREM</b>		
How to explain the differences in the <b>properties of QUADRILATERALS</b>		
How to <b>find the areas of triangles/quadrilaterals/regular polygons</b>		
How to determine the <b>angle relationships</b> when <b>parallel lines</b> are involved		
How to use <b>algebra and angle relationships</b> to solve for an <b>unknown (x)</b>		
How to <b>prove that two triangles are congruent (SSS, SAS and ASA)</b>		
How to <b>prove that two triangles are similar (SSS, SAS and AA)</b>		
How to find the <b>unknown side lengths</b> in <b>similar figures</b>		
How to find side lengths using <b>METRIC RELATIONS</b>		
How to read a <b>FREQUENCY TABLE</b>		
How to make and read a <b>STEM AND LEAF PLOT</b>		
How to calculate <b>MEAN DEVIATION</b> (and what it tells you about the data)		
How to calculate <b>PERCENTILE RANK</b> (and what it means)		
How to <b>find a score of place GIVEN PERCENTILE RANK</b>		
How to read a <b>CONTINGENCY TABLE</b>		
How to make and <b>interpret</b> a <b>SCATTER PLOT</b>		
How to estimate the <b>CORRELATION COEFFICIENT</b> (and what it means)		
How to determine the <b>STRENGTH AND DIRECTION</b> of the <b>CORRELATION COEFFICIENT</b>		
How to determine and represent the <b>EQUATION OF A REGRESSION LINE</b> (e.g. Median-Median method, Meyer line method, best fit method)		
How to <b>draw a curve</b> associated with the chosen model		
How to <b>interpolate or extrapolate values</b> using a <b>REGRESSION LINE</b>		
How to determine the <b>PROBABILITY OF A SINGLE EVENT</b>		
How to determine the <b>PROBABILITY WITH WEIGHTED OUTCOMES</b>		
How to switch back and forth between <b>PROBABILITY AND ODDS</b>		
How to determine <b>ODDS FOR</b> or <b>ODDS AGAINST</b>		
How to <b>CALCULATE MATHEMATICAL EXPECTATION IN GAMES OF CHANCE</b>		
How to <b>MAKE A GAME FAIR</b> using Mathematical expectation		
How to recognize and associate the <b>type of probability</b> to a situation: <b>EXPERIMENTAL, THEORETICAL and SUBJECTIVE</b>		

# **1.1 Points and Segments in the Cartesian Plane**

**Question:**

What is the slope of line segment AB?



- A)  $-2$
- B)  $-\frac{1}{2}$
- C)  $\frac{1}{2}$
- D)  $2$

**General Strategies:**

1. Read the question.
2. Highlight key words.
3. Identify the math topic.
4. Re-read the question.
5. Refer to your memory aid, as needed.
6. Solve the problem without looking at choices shown (A, B, C and D).
7. Look at all the choices.
8. Match your answer to the appropriate choice.

Do not leave a blank!  
Make a choice!

**My Strategies:**



**Answer and Solution:**

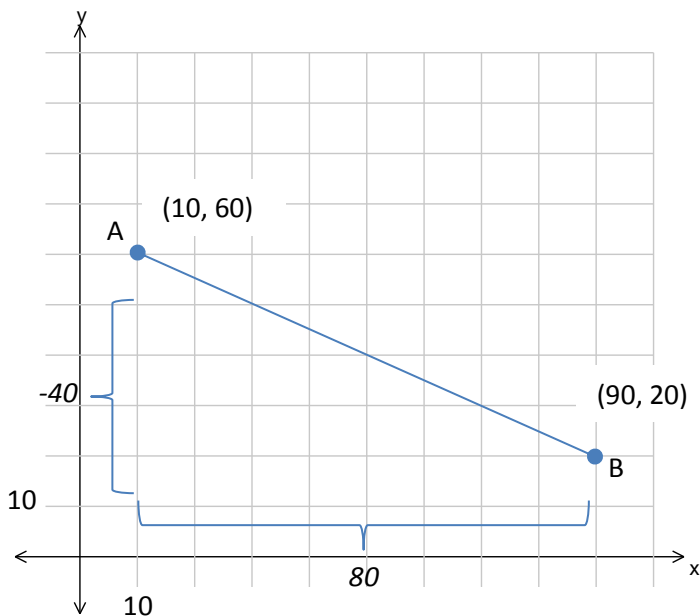
Find the slope between the two points either with the formula or visually from the graph:

- Formula from a table of values (or just the points):

x	y
10	60
90	20

$$\frac{20 - 60}{90 - 10} = \frac{-40}{80} = -\frac{1}{2}$$

- Rise over run from the diagram:  $\frac{rise}{run} = \frac{-40}{80} = -\frac{1}{2}$



- A) Incorrect: would be “run over rise”
- B) **Correct**
- C) Incorrect: Segment AB has a negative slope but  $\frac{1}{2}$  indicates a positive slope
- D) Incorrect: This is the positive version of “run over rise” so incorrect on two counts.

The answer is B.

**Suggested Strategies:**

- I) Recognize the table of values is a series of coordinates... you need two sets to find the rule.
- II) Choose any coordinate pair and label them  $x_1, y_1$  and  $x_2, y_2$
- III) Find the slope between these coordinate pairs using the formula:  $slope = \frac{y_2 - y_1}{x_2 - x_1}$
- IV) Plug the slope into the formula  $y = ax + b$
- V) Substitute any of the  $(x, y)$  coordinate pairs from the table into the equation and solve for the initial value ( $b$ )
- VI) If your slope or initial value is in fraction form, multiply each term by the LCM (least common multiple) of the two denominators
- VII) Keeping in mind the signs, move all of the terms to one side of the equal sign.

**Additional Resources:**

Visions Volume 1, Section 1.1, p. 15 (Slope of a Segment)

Khan Academy video: <http://www.khanacademy.org/math/cc-eighth-grade-math/cc-8th-relationships-functions/cc-8th-slope/v/slope-of-a-line>

Explore Learning Gizmos, <http://www.explorelearning.com/> look up: Slope

**Question:**

What is the endpoint of a line segment which has one end at (6, 18) and the midpoint at (18, 30)?

- A) (-6, 6)
- B) (12, 24)
- C) (24, 12)
- D) (30, 42)

**General Strategies:**

1. Read the question.
2. Highlight key words.
3. Identify the math topic.
4. Re-read the question.
5. Refer to your memory aid, as needed.
6. Solve the problem without looking at choices shown (A, B, C and D).
7. Look at all the choices.
8. Match your answer to the appropriate choice.

Do not leave a blank!  
Make a choice!

**My Strategies:**

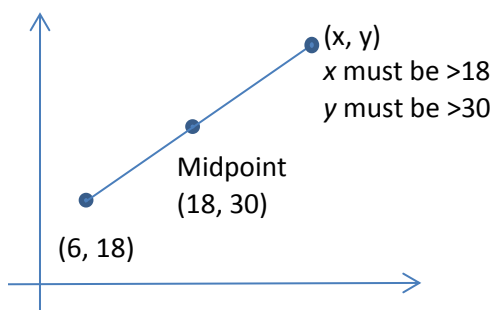
**Answer and Solution:**

Midpoint formula:  $(18, 30): \left(\frac{6+x_2}{2}, \frac{18+y_2}{2}\right)$

$$\begin{array}{rcl} 18 & = & \frac{6+x}{2} \\ 36 & = & 6+x \\ 30 & = & x \end{array} \qquad \begin{array}{rcl} 30 & = & \frac{18+y}{2} \\ 60 & = & 18+y \\ 42 & = & y \end{array}$$

The answer is (30, 42)

OR



The only answer that meets both conditions is D (30, 42).

- A) (–6, 6) This is the answer you get if you mix up the endpoint and the midpoint.
- B) (12, 24) This is the answer you get if you just plug the points into the midpoint formula.
- C) (24, 12) This is if you just plug in the points into the midpoint formula and mix up the x and y.
- D) **(30, 42) This is the correct answer.**

**Suggested Strategies:**

Use either the mid-point formula, remembering that you will have some algebra to do, or sketch it and see which answer makes sense.

**Additional Resources:**

Visions Volume 1, Section 1.1, p. 16 (Point of Division)

Khan Academy video: [http://www.khanacademy.org/math/algebra/linear-equations-and-inequalitie/more-analytic-geometry/e/midpoint\\_formula](http://www.khanacademy.org/math/algebra/linear-equations-and-inequalitie/more-analytic-geometry/e/midpoint_formula)

**Question:**

Point P is located  $\frac{3}{5}$  of the distance from point B (25, 75) and point A (10, 30).

Which coordinates represent point P?

- A) (15.625, 46.875)
- B) (16, 48)
- C) (19, 57)
- D) (19.375, 58.125)

**General Strategies:**

1. Read the question.
2. Highlight key words.
3. Identify the math topic.
4. Re-read the question.
5. Refer to your memory aid, as needed.
6. Solve the problem without looking at choices shown (A, B, C and D).
7. Look at all the choices.
8. Match your answer to the appropriate choice.

Do not leave a blank!  
Make a choice!

**My Strategies:**

**Answer and Solution:**

Division point formula:

$$(x_p, y_p) = \left( x_1 + \frac{a}{b}(x_2 - x_1), y_1 + \frac{a}{b}(y_2 - y_1) \right)$$

With  $a = 3, b = 5, x_1 = 25, y_1 = 75, x_2 = 10, y_2 = 30$

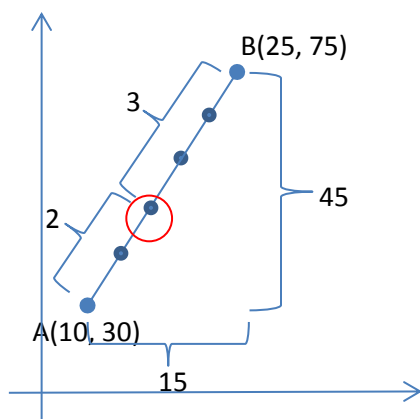
OR  $a = 2, b = 5, x_1 = 10, y_1 = 30, x_2 = 25, y_2 = 75$

Example for the x-coordinate with the first choice:

$$\begin{aligned} x_p &= 25 + \frac{3}{5}(10 - 25) \\ x_p &= 25 + \frac{3}{5}(-15) \\ x_p &= 25 - 9 = 16 \end{aligned}$$

Repeat for the y-coordinate.

Here is the sketch:



- A) Incorrect: this is the answer you get if you use  $\frac{3}{8}$  instead of  $\frac{3}{5}$ .
- B) Correct**
- C) Incorrect: this is the answer you get if you calculate the ratio from the wrong end (A to B instead of B to A).
- D) Incorrect: this is the answer you get if you use  $\frac{3}{8}$  instead of  $\frac{3}{5}$  and went from the wrong end.

**The answer is B.**

**Suggested Strategies:**

- Be careful because the problem is stating the distance from B to A, not A to B. You can see that the distractors assume you might make this mistake.
- Determine whether the ratio given is part to part or part to whole. In this case it is part to whole. You can see that the distractors assume you might make the mistake of interpreting it as a part to part ratio.

You can solve the problem by using the distance formula – paying close attention to where you plug in your points. Remember the B to A. You could also use the other part of the ratio and use  $\frac{2}{5}$  of the way from A to B. Don't let yourself be confused.

You can also sketch the points and see which answer(s) make sense.

**Additional Resources:**

Visions Volume 1, Section 1.1, p. 16 (Point of Division)

Khan Academy video: [http://www.khanacademy.org/math/algebra/linear-equations-and-inequalitie/more-analytic-geometry/e/midpoint\\_formula](http://www.khanacademy.org/math/algebra/linear-equations-and-inequalitie/more-analytic-geometry/e/midpoint_formula)

**Question:**

On a coordinate plane, Jim's house is situated on a line that runs from his school to the swimming pool.

The school is at point A (200, 800) and the pool is at point B (1200, 1600).

Jim's house divides line segment AB into a ratio of 4:1 from point A.

What are the coordinates of Jim's house?

Jim's house is situated at (     ,     ).

**General Strategies:**

1. Read the question.
2. Highlight key words.
3. Identify the math topic.
4. Re-read the question.
5. Make a prediction about the answer- what will it look like? (an equation, a number, etc.).
6. Refer to your memory aid, as needed.
7. Solve.
8. Ask yourself whether your answer makes sense.
9. Write your answer.  
Do not leave a blank!

**My Strategies:**

**Answer and Solution:**

Division point formula:

$$(x_p, y_p) = \left( x_1 + \frac{\text{part}}{\text{whole}}(x_2 - x_1), y_1 + \frac{\text{part}}{\text{whole}}(y_2 - y_1) \right)$$

With  $\frac{\text{part}}{\text{whole}} = \frac{4}{4+1} = \frac{4}{5}$

And  $x_1 = 200, y_1 = 800, x_2 = 1200, y_2 = 1600$

$$x_p = 200 + \frac{4}{5}(1200 - 200)$$

$$x_p = 200 + \frac{4}{5}(1000)$$

$$x_p = 200 + 800$$

$$x_p = 1000$$

$$y_p = 800 + \frac{4}{5}(1600 - 800)$$

$$y_p = 800 + \frac{4}{5}(800)$$

$$y_p = 800 + 640$$

$$y_p = 1440$$

**Jim's house is situated at (1000, 1440).**

**Suggested Strategies:**

Notice the keywords to see what kind of problem it is:

- Line, point
- Divides
- Ratio
- Coordinates

This is a division point question.

Determine whether the ratio given is part to part or part to whole. In this case it is part to part – which means you'll add the numbers to create the fraction in the formula.

Pay attention to the end from which the ratio is being measured – in this case from point A.

It always helps to make a sketch of the situation.

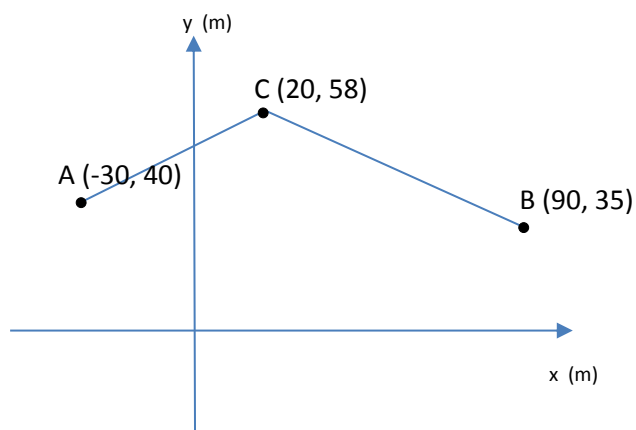
**Additional Resources:**

Visions Volume 1, Section 1.1, p. 16 (Point of Division)

Khan Academy video: [http://www.khanacademy.org/math/algebra/linear-equations-and-inequalitie/more-analytic-geometry/e/midpoint\\_formula](http://www.khanacademy.org/math/algebra/linear-equations-and-inequalitie/more-analytic-geometry/e/midpoint_formula)

**Question:**

How much longer is  $\overline{BC}$  than  $\overline{AC}$ ?



$\overline{BC}$  is \_\_\_\_\_ m longer than  $\overline{AC}$ .

**General Strategies:**

1. Read the question.
  2. Highlight key words.
  3. Identify the math topic.
  4. Re-read the question.
  5. Make a prediction about the answer- what will it look like? (an equation, a number, etc.).
  6. Refer to your memory aid, as needed.
  7. Solve.
  8. Ask yourself whether your answer makes sense.
  9. Write your answer.
- Do not leave a blank!

**My Strategies:**



**Answer and Solution:**

$$d(A, C) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

A (-30,40) & C (20, 58)

$$d(A, C) = \sqrt{(20 - (-30))^2 + (58 - 40)^2}$$

$$d(A, C) = \sqrt{(50)^2 + (18)^2}$$

$$d(A, C) = \sqrt{2500 + 324}$$

$$d(A, C) = \sqrt{2824}$$

$$d(A, C) \approx 53.1413 \text{ m}$$

B (90,35) & C (20, 58)

$$d(B, C) = \sqrt{(20 - 90)^2 + (58 - 35)^2}$$

$$d(B, C) = \sqrt{(-70)^2 + (23)^2}$$

$$d(B, C) = \sqrt{4900 + 529}$$

$$d(B, C) = \sqrt{5429}$$

$$d(B, C) \approx 73.6817 \text{ m}$$

$$73.6817 - 53.1413 = 20.5404 \text{ m}$$

$\overline{BC}$  is 20.54 m longer than  $\overline{AC}$ .

**Suggested Strategies:**

The key word here is “longer” which implies length. And with the Cartesian plane (coordinates) as part of the question we’ll want to use the distance formula.

Determine the distances we need: AC and BC (we don’t need AB).

And then subtract to find the difference between the two distances calculated.

**Additional Resources:**

Visions Volume 1, Section 1.1, p. 15 (Distance between Two Points)

Khan Academy video: <http://www.khanacademy.org/math/algebra/linear-equations-and-inequalitie/more-analytic-geometry/v/midpoint-formula>

**Question:**

Bill says that his house is exactly the same distance to the water tower as Alan's house is.

Alan does not believe him so he makes a Cartesian plane and puts all the information that he knows is true on the graph.

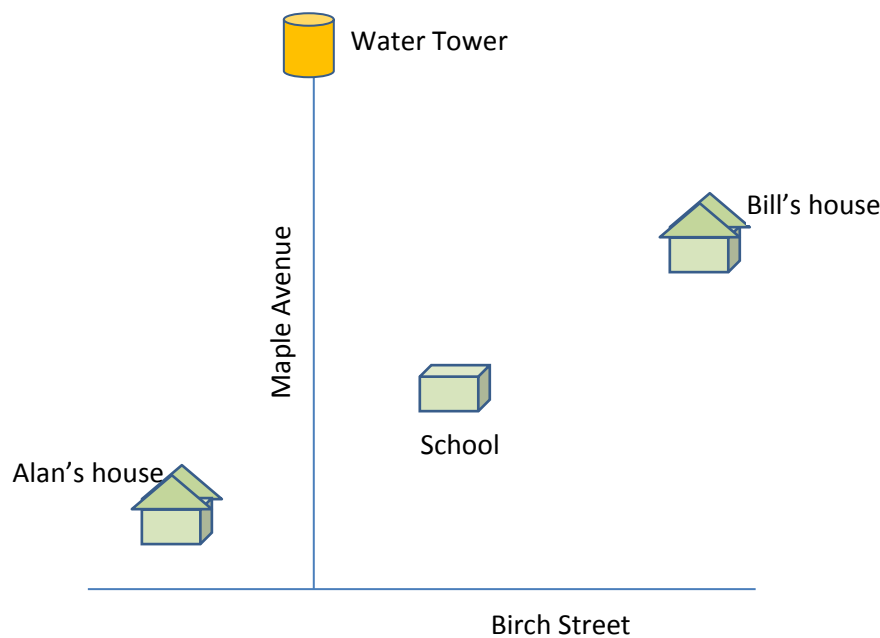
He starts by making Birch St. the x-axis and Maple Ave. the y-axis since they are perpendicular to each other.

He knows his house is in a straight line with Bill's and the school is midway on the line between their houses.

He also knows that the water tower is on Maple Ave. 1100 m from Birch St.

Finally he puts the co-ordinates of his house  $(-400, 200)$  and the co-ordinates of the school  $(200, 400)$  on the graph.

**Which of the boys is correct?**

**General Strategies:**

1. Read the problem.
2. Highlight key words.
3. Identify the math topics.
4. Re-read the problem.
5. Define your steps (your game plan) – this is criteria 3.
6. Refer to your memory aid, as needed.
7. Solve.
8. If you get stuck on a calculation, pick a number and keep going.
9. Ask yourself whether your answer makes sense.
10. Write your answer statement.

Show any or all your work! Do not leave a blank page!

**My Strategies:**

**Answer and Solution:**

Distance between Bill's house and the water tower:

Bill's house:  $(-400, 200)$       Water tower:  $(0, 1100)$

$$\begin{aligned}d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\d &= \sqrt{(0 - (-400))^2 + (1100 - 200)^2} \\d &= \sqrt{(400)^2 + (900)^2} \\d &= \sqrt{160000 + 810000} \\d &= \sqrt{970000} \\d &\approx \mathbf{984.88 \text{ m}}\end{aligned}$$

Coordinates of Alan's house:

Bill's house:  $(-400, 200)$

School:  $(200, 400)$ , the midpoint

$$\frac{x_1 + x_2}{2} = x_m \qquad \frac{y_1 + y_2}{2} = y_m$$

$$\frac{-400 + x_2}{2} = 200 \qquad \frac{200 + y_2}{2} = 400$$

$$\begin{aligned}-400 + x_2 &= 400 & 200 + y_2 &= 800 \\x_2 &= 800 & y_2 &= 600\end{aligned}$$

Alan's house:  $(800, 600)$

$$\begin{aligned}d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\d &= \sqrt{(0 - 800)^2 + (1100 - 600)^2} \\d &= \sqrt{(-800)^2 + (500)^2} \\d &= \sqrt{640000 + 250000} \\d &= \sqrt{890000} \\d &\approx \mathbf{943.40 \text{ m}}\end{aligned}$$

**Alan is correct; their houses are not the same distance from the water tower.**

**Suggested Strategies:**

Begin by transferring information from the text onto the diagram.

To answer this question you have to calculate the distance between each of the houses and the water tower. For that, you need the three sets of coordinates; Bill's house, Alan's house and the water tower.

You are given the coordinates of the water tower  $(0, 1100)$  and Bill's house  $(-400, 200)$ .

Using the coordinates of Bill's house and the school, you can determine the coordinates of Alan's house.

**Additional Resources:**

Visions Volume 1, Section 1.1, pp. 15-16

Khan Academy video: <http://www.khanacademy.org/math/algebra/linear-equations-and-inequalitie/more-analytic-geometry/v/midpoint-formula>



## **1.2 Lines in the Cartesian Plane**

**Question:**

What is the rule for the linear function that corresponds to the table below?

$x$	-10.2	-6.4	3.4	12.8
$y$	53.7	40.4	6.1	-26.8

- A)  $-7x - 2y - 36 = 0$
- B)  $7x - 2y + 36 = 0$
- C)  $-7x + 2y - 36 = 0$
- D)  $7x + 2y - 36 = 0$

**General Strategies:**

1. Read the question.
2. Highlight key words.
3. Identify the math topic.
4. Re-read the question.
5. Refer to your memory aid, as needed.
6. Solve the problem without looking at choices shown (A, B, C and D).
7. Look at all the choices.
8. Match your answer to the appropriate choice.

Do not leave a blank!  
Make a choice!

**My Strategies:**

**Answer and Solution:**

Formula for a linear equation:  $y = ax + b$

Coordinates used to find the rule:

(-10.2, 53.7)

(-6.4, 40.4)

$$\text{slope } (a) = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\text{slope } (a) = \frac{40.4 - 53.7}{-6.4 - (-10.2)} = \frac{-13.3}{3.8} = -3.5$$

$$y = ax + b$$

$$y = -3.5x + b$$

$$40.4 = -3.5(-6.4) + b$$

$$40.4 = 22.4 + b$$

$$18 = b$$

$$y = -3.5x + 18$$

Convert slope from decimal form to fraction form and multiply each term by the denominator

$$y = \frac{-7}{2}x + 18$$

$$2y = -7x + 36$$

$$2y + 7x - 36 = 0$$

$$7x + 2y - 36 = 0$$

The answer is D.

**Suggested Strategies:**

- I) Recognize the table of values is a series of coordinates... you need two sets to find the rule.
- II) Choose any coordinate pair and label them  $x_1$ ,  $y_1$  and  $x_2$ ,  $y_2$
- III) Find the slope between these coordinate pairs using the formula:  $\text{slope} = \frac{y_2 - y_1}{x_2 - x_1}$
- IV) Plug the slope into the formula  $y = ax + b$
- V) Substitute any of the (x, y) coordinate pairs from the table into the equation and solve for the initial value (b)
- VI) If your slope or initial value is in fraction form, multiply each term by the LCM (least common multiple) of the two denominators
- VII) Keeping in mind the signs, move all of the terms to one side of the equal sign.

**Additional Resources:**

Visions Volume 1, Section 1.2, p. 26 (Equation of Line from Slope and Intercepts)

Khan Academy video: <http://www.khanacademy.org/math/algebra/linear-equations-and-inequalitie/more-analytic-geometry/v/algebra--slope-and-y-intercept-intuition>

Explore Learning Gizmos, <http://www.explorellearning.com/> look up:

- Point-Slope Form of a Line
- Slope-Intercept Form of a Line

**Question:**

Which of the following equations represents a line perpendicular to  $4x + 3y + 12 = 0$ ?

- A)  $3x + 4y - 8 = 0$
- B)  $y = \frac{4}{3}x - 4$
- C)  $-3x + 4y - 8 = 0$
- D)  $y = -\frac{4}{3}x + 4$

**General Strategies:**

1. Read the question.
2. Highlight key words.
3. Identify the math topic.
4. Re-read the question.
5. Refer to your memory aid, as needed.
6. Solve the problem without looking at choices shown (A, B, C and D).
7. Look at all the choices.
8. Match your answer to the appropriate choice.

Do not leave a blank!  
Make a choice!

**My Strategies:**



**Answer and Solution:**

Find the slope of  $4x + 3y + 12 = 0$

$$3y = -4x - 12$$

$$\frac{3y}{3} = \frac{-4x - 12}{3}$$

$$y = \frac{-4}{3}x - 4 \quad \text{the slope is } \frac{-4}{3}$$

Find the perpendicular slope:  $\frac{a}{b} \rightarrow \frac{-b}{a}$

The perpendicular slope is  $\frac{3}{4}$  (options B and D are 'out')

Convert options A and C to slope-intercept form and compare slopes.

**Option A**

$$3x + 4y - 8 = 0$$

$$4y = -3x + 8$$

$$\frac{4y}{4} = \frac{-3x + 8}{4},$$

The slope is not  $\frac{3}{4}$ , option A is wrong.

**Option C**

$$-3x + 4y - 8 = 0$$

$$4y = 3x + 8$$

$$\frac{4y}{4} = \frac{3x + 8}{4}$$

The slope is equal to  $\frac{3}{4}$ , **option C is correct.**

**The answer is C.**

**Suggested Strategies:**

- I) The word *perpendicular* in this problem should immediately cause you to write the negative reciprocal rule for perpendicular slopes:  $\frac{a}{b} \rightarrow \frac{-b}{a}$
- II) Start by converting the equation from 'general' form to 'slope-intercept' form in order to get a better look at the slope.
- III) Find the negative reciprocal of the slope from the equation given in the problem. This is the slope we are looking for in our multiple-choice answers.
- IV) Remember that we only care about finding a perpendicular line in this problem, so we only need to worry about the slopes. Ignore the initial values altogether... they are only distractors here.

**Additional Resources:**

Visions Volume 1, Section 1.2, p. 27 (Perpendicular Line)

Khan Academy video: [http://www.khanacademy.org/math/algebra/linear-equations-and-inequalitie/more-analytic-geometry/e/line\\_relationships](http://www.khanacademy.org/math/algebra/linear-equations-and-inequalitie/more-analytic-geometry/e/line_relationships)

**Question:**

What is the  $x$ -intercept for the following linear equation:

$$2x + 3y + 6 = 0$$

- A) -3
- B) -2
- C) 2
- D) 3

**General Strategies:**

1. Read the question.
2. Highlight key words.
3. Identify the math topic.
4. Re-read the question.
5. Refer to your memory aid, as needed.
6. Solve the problem without looking at choices shown (A, B, C and D).
7. Look at all the choices.
8. Match your answer to the appropriate choice.

Do not leave a blank!  
Make a choice!

**My Strategies:**

**Answer and Solution:**

Set the 'y' value to 0 and solve for x.

$$2x + 0 + 6 = 0$$

$$2x + 6 = 0$$

$$2x = -6$$

$$x = \frac{-6}{2}$$

$$x = -3$$

**Suggested Strategies:**

- I) Remember that the x-intercept is the point on a graph where the line crosses the x-axis ( $y = 0$ )
- II) Set  $y = 0$  and solve for x.

The answer is A.

**Additional Resources:**

Visions Volume 1, Section 1.2, p. 26 (Equation of a Line)

Khan Academy video: <http://www.khanacademy.org/math/algebra/linear-equations-and-inequalitie/more-analytic-geometry/v/algebra--equation-of-a-line>

**Question:**

What is the equation of the line parallel to  $3x - 4y - 24 = 0$  that passes through point  $P(-8, 7)$ ?

- A)  $-3x + 4y - 13 = 0$
- B)  $-3x + 4y - 52 = 0$
- C)  $y = \frac{3x}{4} + 52$
- D)  $3y = -4x - 13$

**General Strategies:**

1. Read the question.
2. Highlight key words.
3. Identify the math topic.
4. Re-read the question.
5. Refer to your memory aid, as needed.
6. Solve the problem without looking at choices shown (A, B, C and D).
7. Look at all the choices.
8. Match your answer to the appropriate choice.

Do not leave a blank!  
Make a choice!

**My Strategies:**

**Answer and Solution:**

Convert the equation from general form to y-intercept form

$$3x - 4y - 24 = 0$$

$$-4y = -3x + 24$$

$$\frac{-4y}{-4} = \frac{-3x + 24}{-4}$$

$$y = \frac{3x}{4} - 6, \text{ the slope of the parallel line must be } \frac{3}{4}$$

$$y = ax + b$$

$$y = \frac{3}{4}x + b$$

$$7 = 0.75(-8) + b$$

$$7 = -6 + b$$

$$13 = b$$

$$y = \frac{3}{4}x + 13$$

Convert the rule from slope-intercept form to general form by multiplying by the denominator

$$(4)y = (4)\frac{3}{4}x + (4)13$$

$$4y = 3x + 52$$

$$4y - 3x - 52 = 0$$

$$-3x + 4y - 52 = 0$$

The answer is B.

**Suggested Strategies:**

- I) Remember that parallel lines always have the same slope.
- II) Convert the rule in the question from 'general' form to 'slope-intercept' form in order to find the slope. This slope will be the same in your new parallel line.
- III) Plug the parallel slope (a) into the formula  $y = ax + b$
- IV) Substitute the coordinates of point P (-8, 7) into the new equation and solve for the initial value (b).
- V) Convert your answer from slope-intercept form to general form and look for a match.
- VI) You can also convert the rules from general form into slope-intercept form in the multiple choice section.

**Additional Resources:**

Visions Volume 1, Section 1.2, p. 27 (Parallel Line)

Khan Academy video: <http://www.khanacademy.org/math/algebra/linear-equations-and-inequalitie/more-analytic-geometry/v/parallel-line-equation>

**Question:**

Line  $L_1$  passes P (12, 15) and is perpendicular to line  $L_2$   
Line  $L_2$  has equation  $-4x + 5y - 10 = 0$ .

What is the  $x$ -intercept of the line  $L_1$ ?

The  $x$ -intercept of the line  $L_1$  is \_\_\_\_\_

**General Strategies:**

1. Read the question.
  2. Highlight key words.
  3. Identify the math topic.
  4. Re-read the question.
  5. Make a prediction about the answer-what will it look like? (an equation, a number, etc.).
  6. Refer to your memory aid, as needed.
  7. Solve.
  8. Ask yourself whether your answer makes sense.
  9. Write your answer.
- Do not leave a blank!

**My Strategies:**

**Answer and Solution:**

Convert the equation line  $L_2$  from general form to standard form

$$-4x + 5y - 10 = 0$$

$$5y = 4x + 10$$

$$\frac{5y}{5} = \frac{4x + 10}{5}$$

$$y = \frac{4x}{5} + 2, \text{ the slope of the line } L_2 \text{ is } \frac{4}{5}$$

Find the perpendicular slope of line  $L_1$ :

$$\frac{a}{b} \rightarrow \frac{-b}{a}, \frac{4}{5} \rightarrow \frac{-5}{4}, \text{ the perpendicular slope is } \frac{-5}{4}$$

$$y = ax + b$$

$$y = \frac{-5}{4}x + b$$

$$15 = -1.2(12) + b$$

$$15 = -14.4 + b$$

$$30 = b$$

$$y = \frac{-5}{4}x + 30$$

Set the 'y' value to 0 and solve for x.

$$0 = \frac{-5}{4}x + 30$$

$$-30 = \frac{-5}{4}x$$

$$-30 \left( \frac{4}{-5} \right) = x$$

$$24 = x$$

The x-intercept of the line  $L_1$  is 24

**Suggested Strategies:**

- I) Remember that an x-intercept is the point at which a line crosses the x-axis. The y-value of this coordinate must be equal to 0 ( $y = 0$ ) at this point.
- II) Start by converting the equation from general form to standard form in order to get a better look at the slope.
- III) The word *perpendicular* in this problem should immediately cause you to write the negative reciprocal rule for perpendicular slopes:  $\frac{a}{b} \rightarrow \frac{-b}{a}$ .
- IV) Since we are looking for the line that is *perpendicular* to  $-4x + 5y - 10$ , we'll need the negative reciprocal ( $\frac{-b}{a}$ ) of the slope from the equation given in the problem.
- V) Use the perpendicular slope in a new ' $y = ax + b$ ' rule.
- VI) Substitute the coordinates of point P (12, 15) into the new equation and solve for the initial value ( $b$ ).
- VII) Once you've got your rule for the perpendicular line finished, find the x-intercept by making  $y = 0$  and solving for x.

**Additional Resources:**

Visions Volume 1, Section 1.2, pp. 26-27

Khan Academy video: <http://www.khanacademy.org/math/algebra/linear-equations-and-inequalitie/more-analytic-geometry/v/parallel-line-equation>

**Question:**

Line  $L_1$  has a slope of  $\frac{4}{3}$  and a  $y$ -intercept of  $-3$ .

Line  $L_2$  is perpendicular to line  $L_1$  and passes through point  $(2, 5)$ .

What is the equation of line  $L_2$ ?

The equation of line  $L_2$  is \_\_\_\_\_

**General Strategies:**

1. Read the question.
  2. Highlight key words.
  3. Identify the math topic.
  4. Re-read the question.
  5. Make a prediction about the answer- what will it look like? (an equation, a number, etc.).
  6. Refer to your memory aid, as needed.
  7. Solve.
  8. Ask yourself whether your answer makes sense.
  9. Write your answer.
- Do not leave a blank!

**My Strategies:**



**Answer and Solution:**Line  $L_1$  :

$$y = \frac{4x - 3}{3}$$

Line  $L_2$  :

$$a = \frac{-3}{4}$$

Substitute point (2, 5) in for the x and y to solve for "b".

$$y = \frac{-3x}{4} + b$$

$$y = \frac{-3(2)}{4} + b$$

$$5 = \frac{-6}{4} + b$$

$$b = 5 + \frac{3}{2} = 6.5$$

$$y = \frac{-3x}{4} + 6.5$$

The equation of line  $L_2$  is  $y = \frac{-3x}{4} + 6.5$ **Suggested Strategies:**

- Use the negative reciprocal of the slope of line  $L_1$  to find the slope of line  $L_2$ .

- Use the function form of the equation,  $y = ax + b$ , with point (2, 5) and the new slope.

**Additional Resources:**

Visions Volume 1, p. 27 (Mathematical Knowledge Summary)

Khan Academy video: <http://www.khanacademy.org/math/algebra/systems-of-eg-and-ineq/fast-systems-of-equations/v/solving-systems-of-equations-by-elimination>

Explore Learning Gizmos, <http://www.explorelearning.com/> look up: Systems of Linear Equations

**Question:**

Given the equation:

$$8x + 6y + 12 = 0$$

- A) What is the  $x$ -intercept?
- B) What is the  $y$ -intercept?

The  $x$ -intercept is \_\_\_\_\_

The  $y$ -intercept is \_\_\_\_\_

**General Strategies:**

1. Read the question.
  2. Highlight key words.
  3. Identify the math topic.
  4. Re-read the question.
  5. Make a prediction about the answer- what will it look like? (an equation, a number, etc.).
  6. Refer to your memory aid, as needed.
  7. Solve.
  8. Ask yourself whether your answer makes sense.
  9. Write your answer.
- Do not leave a blank!

**My Strategies:**

**Answer and Solution:****Algebraically:**

A) x-intercept

$$y = 0$$

$$8x + 6y + 12 = 0$$

$$8x + 6(0) + 12 = 0$$

$$8x = -12$$

$$x = \frac{-12}{8} = \frac{-3}{2}$$

$$x = \frac{-3}{2}$$

B) y-intercept

$$x = 0$$

$$8(0) + 6y + 12 = 0$$

$$6y + 12 = 0$$

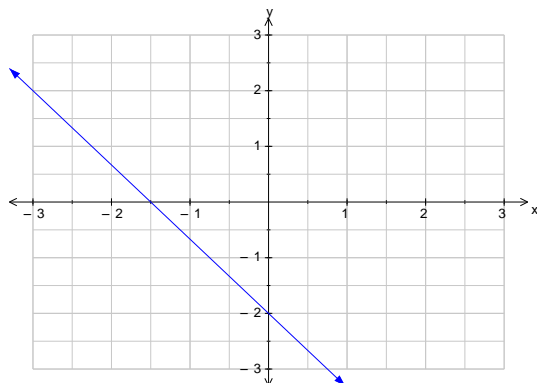
$$6y = -12$$

$$y = \frac{-12}{6} = -2$$

$$y = -2$$

**Graphically:**

$$8x + 6y + 12 = 0 \rightarrow y = -\frac{4}{3}x - 2$$



The x-intercept is  $-\frac{3}{2}$ .

The y-intercept is  $-2$ .

**Suggested Strategies:**

When finding the intercepts, the other coordinate is 0:

x-intercept means  $y = 0$

y-intercept means  $x = 0$ .

You can also solve this question graphically by changing the equation into function form and plotting the y-intercept and slope.

**Additional Resources:**

Visions Volume 1, p. 26 (Mathematical Knowledge Summary)

Explore Learning Gizmos, <http://www.explorelearning.com/> look up:

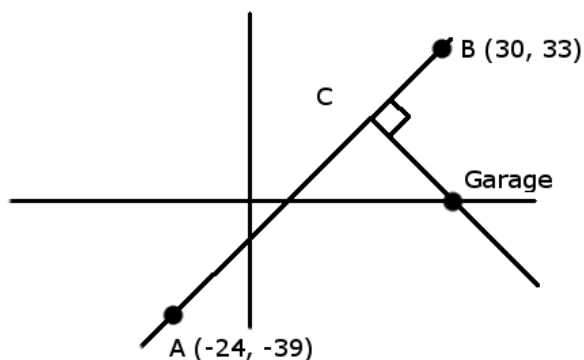
**Question:**

A car is travelling along a straight path from point A  $(-24, -39)$  to point B  $(30, 33)$ .

The car breaks down, having completed exactly two-thirds of the trip.

A tow truck must travel along a path that is perpendicular to the car's path, leaving from a garage located somewhere along the x-axis.

How far must the tow-truck travel to get from the garage to the car?  
(All units are in km.)

**General Strategies:**

1. Read the problem.
2. Highlight key words.
3. Identify the math topics.
4. Re-read the problem.
5. Define your steps (your game plan) – this is criteria 3.
6. Refer to your memory aid, as needed.
7. Solve.
8. If you get stuck on a calculation, pick a number and keep going.
9. Ask yourself whether your answer makes sense.
10. Write your answer statement.

Show any or all your work! Do not leave a blank page!

**My Strategies:**

**Answer and Solution:**

Find the coordinates of the point where the car breaks down

Division point  $\frac{2}{3}$  of the way between A and B

$$\text{Division point} : \left( x_1 + \frac{a}{b}(x_2 - x_1), y_1 + \frac{a}{b}(y_2 - y_1) \right)$$

**A (-24, -39)**

**B (30, 33)**

$$\left( -24 + \frac{2}{3}(30 - (-24)), -39 + \frac{2}{3}(33 - (-39)) \right)$$

$$\left( -24 + \frac{2}{3}(54), -39 + \frac{2}{3}(72) \right)$$

**Call the position of the car point C**

**C (12, 9)**

Find the slope of the rule for the line the car travels

**Formula for a linear equation:  $y = ax + b$**

*Coordinates used to find the rule:*

**A (-24, -39)**

**B (30, 33)**

$$\text{slope } (a) = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\text{slope } (a) = \frac{33 - (-39)}{30 - (-24)} = \frac{72}{54} = \frac{4}{3}$$

Find the equation for the tow-truck's path

The tow-truck's path will be perpendicular (negative reciprocal slope) and passes through point C (12, 9), *the car*

$$\frac{a}{b} \rightarrow \frac{-b}{a}, \frac{4}{3} \rightarrow \frac{-3}{4}, \text{ the tow-truck's slope is } \frac{-3}{4}$$

**$y = ax + b$ , passing through (12, 9)**

$$y = -\frac{3}{4}x + b$$

$$9 = \frac{-3}{4}(12) + b$$

$$9 = -9 + b$$

$$18 = b$$

**Suggested Strategies:**

- I) Recognize that the question is asking for a distance between two points, the car and the garage. This problem requires us to first find and then use those coordinates.
- II) Start by using the division point formula to find the coordinates of the car when it breaks down.
- III) Then find the coordinates of the garage,
  - a. we know that it is on the x-axis (so the y-coordinate is = 0)
  - b. We know that it is on the path that is perpendicular to AB
- IV) Find the slope of AB so we can use its negative reciprocal to define the slope of the line between the car and the garage.
- V) Plug the (x, y) coordinates of the car into the formula for the tow-truck's path, then solve for the initial value to complete the equation for the tow-truck's path.

$$y = \frac{-3}{4}x + 18$$

Find the coordinates of the Garage on the x-axis

Set the 'y' value to 0 and solve for x.

$$y = \frac{-3}{4}x + 18$$

$$0 = \frac{-3}{4}x + 18$$

$$-18 = \frac{-3}{4}$$

$$-18 \left( \frac{4}{-3} \right) = x$$

$$24 = x$$

**The coordinates of the garage are: (24, 0)**

Find the distance from the garage to the car.

**Garage (24, 0)**

**Car (12, 9)**

$$distance = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$distance = \sqrt{(12 - 24)^2 + (9 - 0)^2}$$

$$distance = \sqrt{(-12)^2 + (9)^2}$$

$$distance = \sqrt{144 + 81}$$

$$distance = 15$$

**Final answer: the distance the tow-truck must travel from the garage to the car is 15 km.**

VI) Using the equation of the line for the tow-truck's path, set the y-coordinate to 0 (since the garage is on the x-axis) and solve for x.

VII) Use the distance formula along with the coordinates of the car and garage to find your final answer.

### **Additional Resources:**

Visions Volume 1, Section 1.2, pp. 26-27

Visions Volume 1, Section 1.1, pp. 15-16

## **1.3 Systems of Equations**

**Question:**

Consider the following system of linear equations.

$$y = -\frac{2}{3}x - 2$$

$$y = -\frac{2}{3}x - 4$$

Which of the following statements is true?

- A) The system has an infinite number of solutions.
- B) The system has a unique solution.
- C) The system has no solution.
- D) The system has two solutions.

**General Strategies:**

1. Read the question.
2. Highlight key words.
3. Identify the math topic.
4. Re-read the question.
5. Refer to your memory aid, as needed.
6. Solve the problem without looking at choices shown (A, B, C and D).
7. Look at all the choices.
8. Match your answer to the appropriate choice.

Do not leave a blank!  
Make a choice!

**My Strategies:**



**Answer and Solution:**

The slopes ( $a$ ) are the same and the  $y$ -intercepts ( $b$ ) are different; if graphed you would see that the lines are parallel and never intersect.

**The answer is C.**

**Suggested Strategies:**

Solve the system.

Check to see if the slopes are the same;

- If not, there will be one solution;
- If the slopes are the same, check to see if the  $y$ -intercepts are the same
  - If they are the same, there is an infinite number of solutions since they are the same line;
  - If they are not, they are parallel lines and the system has no solution.

**Additional Resources:**

Visions Volume 1, p. 40 (Mathematical Knowledge Summary)

Khan Academy video: <http://www.khanacademy.org/math/algebra/systems-of-eq-and-ineq/fast-systems-of-equations/v/solving-systems-of-equations-by-elimination>

Explore Learning Gizmos, <http://www.explorelearning.com/> look up: Systems of Linear Equations

**Question:**

Given the following system of equations.

$$2x - 5y + 12 = 0$$

$$x - 3y = 4$$

What is the solution for this system?

- A)  $(-41, -14)$
- B)  $(-44, -20)$
- C)  $(-56, -20)$
- D)  $(-56, -12)$

**General Strategies:**

1. Read the question.
2. Highlight key words.
3. Identify the math topic.
4. Re-read the question.
5. Refer to your memory aid, as needed.
6. Solve the problem without looking at choices shown (A, B, C and D).
7. Look at all the choices.
8. Match your answer to the appropriate choice.

Do not leave a blank!  
Make a choice!

**My Strategies:**

**Answer and Solution:**

$$2x - 5y + 12 = 0$$

$$x - 3y = 4$$

Substitution method (since it is easy to isolate x.)

$$x - 3y = 4$$

$$x = 3y + 4$$

$$2x - 5y + 12 = 0$$

$$2(3y + 4) - 5y + 12 = 0$$

$$6y + 8 - 5y + 12 = 0$$

$$y + 20 = 0$$

$$y = -20$$

$$x = 4 + 3y$$

$$x = 4 + 3(-20)$$

$$x = 4 - 60$$

$$x = -56$$

$$(-56, -20)$$

Check:

$$2x - 5y + 12 = 0$$

$$2(-56) - 5(-20) + 12 = 0$$

$$-112 + 100 + 12 = 0 \quad \text{True}$$

$$x - 3y = 4$$

$$(-56) - 3(-20) = 4$$

$$-56 + 60 = 4 \quad \text{True}$$

Both are true, so  $(-56, -20)$  is the correct solution.

**The answer is C.**

**Suggested Strategies:**

Method 1:

Solve the system of equations by the method of your choice – this one lends itself to substitution.

Method 2:

Check by substituting each possible answer into the two equations to verify which point is a possible solution.

$$(-41, -14)$$

$$2x - 5y + 12 = 0$$

$$2(-41) - 5(-14) + 12 = 0$$

$$-82 + 70 + 12 = 0$$

$$0 = 0$$

$$x - 3y = 4$$

$$-41 - 3(-14) = 4$$

$$-41 + 42 = 4$$

$$11 \neq 4$$

$$(-44, -20)$$

$$2x - 5y + 12 = 0$$

$$2(-44) - 5(-20) + 12 = 0$$

$$-88 + 100 + 12 = 0$$

$$24 \neq 0$$

$$(-56, -12)$$

$$2x - 5y + 12 = 0$$

$$2(-56) - 5(-12) + 12 = 0$$

$$-112 + 60 + 12 = 0$$

$$-40 \neq 0$$

**Additional Resources:**

Visions Volume 1, p. 39 (Mathematical Knowledge Summary)

Khan Academy video: <http://www.khanacademy.org/math/algebra/systems-of-eg-and-ineq/fast-systems-of-equations/v/solving-systems-of-equations-by-elimination>

Explore Learning Gizmos, <http://www.explorelearning.com/> look up: Systems of Linear Equations

**Question:**

Stephane purchased 3 chocolate chip cookies and 4 peanut butter cookies for \$5.65. Marie purchased 5 chocolate chip cookies and 7 peanut butter cookies for \$9.70.

What is the price of a peanut butter cookie?

- A) 70 cents
- B) 75 cents
- C) 80 cents
- D) 85 cents

**General Strategies:**

1. Read the question.
2. Highlight key words.
3. Identify the math topic.
4. Re-read the question.
5. Refer to your memory aid, as needed.
6. Solve the problem without looking at choices shown (A, B, C and D).
7. Look at all the choices.
8. Match your answer to the appropriate choice.

Do not leave a blank!  
Make a choice!

**My Strategies:**

**Answer and Solution:**

$x$  = cost of a chocolate chip cookie

$y$  = cost of a peanut butter cookie

$$3x + 4y = 5.65$$

$$5x + 7y = 9.70$$

$$-5(3x + 4y = 5.65)$$

$$3(5x + 7y = 9.70)$$

$$-15x - 20y = -28.25$$

$$\underline{15x + 21y = 29.10}$$

$$y = 0.85$$

cost of a peanut butter cookie = \$0.85

- A) 70 cents
- B) 75 cents – is cost of chocolate chip cookies
- C) 80 cents
- D) **85 cents**

The answer is D.

**Suggested Strategies:**

Set up a system of equations and solve it.

- Define your variables
- Write your equations
- Choose a method (this one suggests elimination method but the other methods work as well.)
- Interpret your answer correctly by seeing which variable represents the cost of the peanut butter cookie.

**Additional Resources:**

Visions Volume 1, pp. 39-40 (Mathematical Knowledge Summary)

Khan Academy video: <http://www.khanacademy.org/math/algebra/systems-of-eq-and-ineq/fast-systems-of-equations/v/solving-systems-of-equations-by-elimination>

Explore Learning Gizmos, <http://www.explorelearning.com/> look up: Systems of Linear Equations

**Question:**

A kitchen cabinetmaker has two models of upper cupboards which a client can choose from to complete a kitchen installation.

Three different clients ordered different combinations of tall and short cabinets. The total cost including delivery is listed below for clients A and B.

Client	Number of Tall Cabinets	Number of Short Cabinets	Delivery Cost	Total Cost
A	7	4	\$120	\$1840
B	9	8	\$190	\$2630
C	11	2	\$170	?

Client C believes his total cost is lower than client B's.

Is he correct?

Yes

No

**General Strategies:**

1. Read the problem.
  2. Highlight key words.
  3. Identify the math topics.
  4. Re-read the problem.
  5. Define your steps (your game plan) – this is criteria 3.
  6. Refer to your memory aid, as needed.
  7. Solve.
  8. If you get stuck on a calculation, pick a number and keep going.
  9. Ask yourself whether your answer makes sense.
  10. Write your answer statement.
- Show any or all your work! Do not leave a blank page!

**My Strategies:**

**Answer and Solution:**

$x$  = cost for long cabinet  
 $y$  = cost for short cabinet

Client A:  $7x + 4y + 120 = 1840$     OR     $7x + 4y = 1720$

Client B:  $9x + 8y + 190 = 2630$     OR     $9x + 8y = 2440$

Client C:  $11x + 2y + 170 = ?$

By elimination method:

Step 1)

$$-2(7x + 4y = 1720)$$

$$9x + 8y = 2440$$

$$-14x - 8y = -3440$$

$$\underline{9x + 8y = 2440}$$

$$-5x = -1000$$

$$x = \frac{-1000}{-5}$$

$$x = 200$$

$$x = 200$$

$$y = 80$$

Client C:  $11x + 2y + 170 = ?$

$$11(200) + 2(80) + 170 =$$

$$2200 + 160 + 170 = 2530$$

\$2530

**Client C is correct. His total cost will be lower than client B's, since it is \$2530 compared to \$2630.**

**Suggested Strategies:**

This is a "system of equations" question.

In order to find the cost for Client C, you need to know how much each type of cabinet costs.

Use the information given for the other two clients to find those costs.

- Define your variables,
- Set up two equations in two unknowns,
- Solve the system,
- Use the solution to find the cost for Client C

Note: if you don't show any work and just check one of the boxes, you will get zero.

**Additional Resources:**

Visions Volume 1, pp. 39-40 (Mathematical Knowledge Summary)

Khan Academy video: <http://www.khanacademy.org/math/algebra/systems-of-eg-and-ineq/fast-systems-of-equations/v/solving-systems-of-equations-by-elimination>

Explore Learning Gizmos, <http://www.explorellearning.com/> look up: Systems of Linear Equations



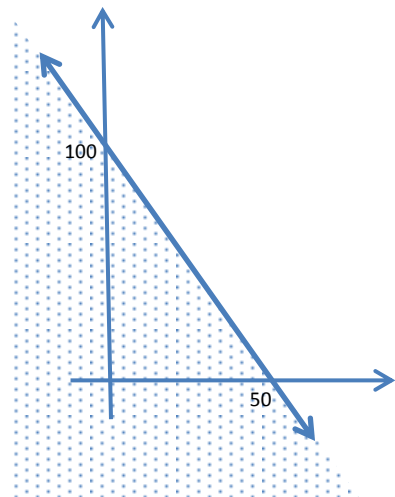


## **1.4 Half-Planes in the Cartesian Plane**

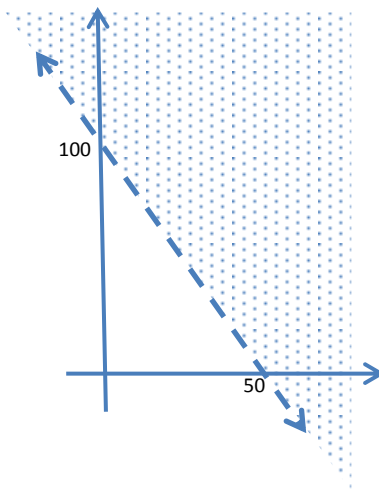
**Question:**

In which graph below is point  $(100, 200)$  a solution of the inequality?

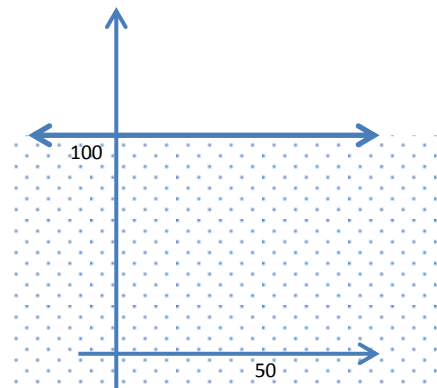
A)



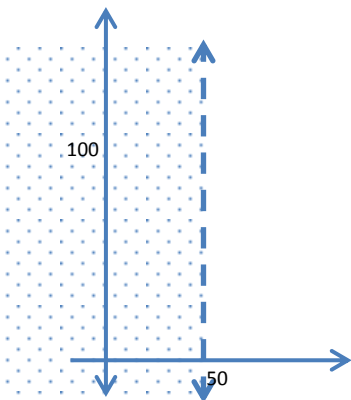
C)



B)



D)



**General Strategies:**

1. Read the question.
2. Highlight key words.
3. Identify the math topic.
4. Re-read the question.
5. Refer to your memory aid, as needed.
6. Look carefully at each choice shown (A, then B, then C and then D).
7. Eliminate options you know to be incorrect.
8. Solve/check each possible choice.
9. Select the choice that makes the most sense.

Do not leave a blank!

Make a choice!

**My Strategies:**

**Answer and Solution:**

Method 1:

Plot the point on each graph and see which one is in the shaded area.

Method 2:

A	$y \leq -2x + 100$	$200 \leq -2(100) + 100$ $200 \leq -100$	False
B	$y \leq 100$	$200 \leq 100$	False
C	$y > -2x + 100$	$200 > -2(100) + 100$ $200 > -100$	True
D	$x < 50$	$100 < 50$	False

- A) Incorrect – the point has an  $x$ -coordinate to the right of 50 and that region is not shaded.
- B) Incorrect – the point has a  $y$ -coordinate that is above 100 and that region is not shaded.
- C) **Correct – the point has an  $x$ -coordinate to the right of 50 and a  $y$ -coordinate above 100 and that region is shaded.**
- D) Incorrect – the point has an  $x$ -coordinate to the right of 50 and that region is not shaded.

The answer is C.

**Suggested Strategies:**

Method 1:

Begin by analyzing visually. Approximately where will point (100,200) be?

Which graph will have that point in its shaded region?

Method 2:

Write the rule for each graph and test the coordinates of the point in each one.

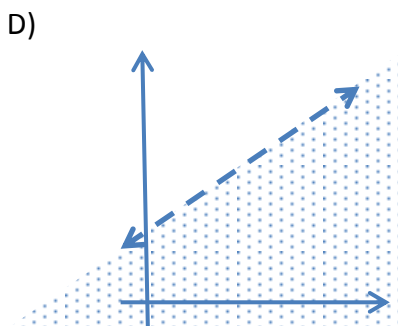
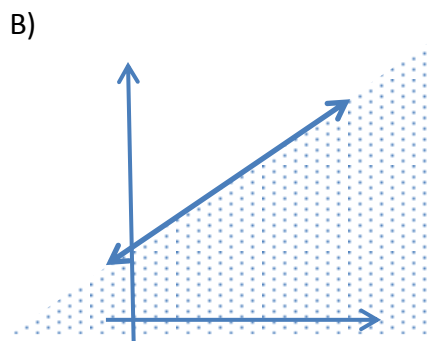
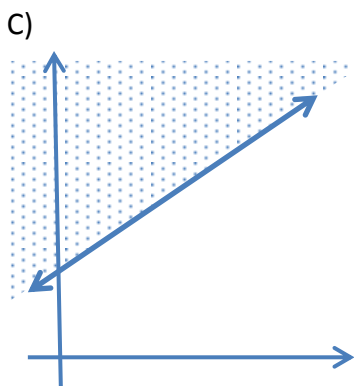
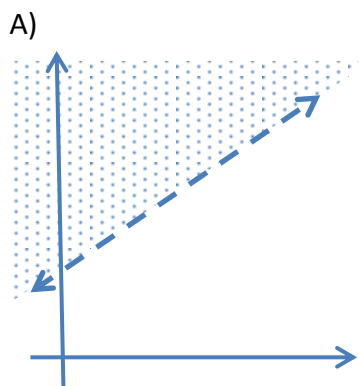
**Additional Resources:**

Visions Volume 1, Section 1.4, pp. 49-50

Khan Academy video: <http://www.khanacademy.org/math/algebra/linear-equations-and-inequalitie/graphing-linear-inequalities/v/graphing-inequalities>

**Question:**

Which graph below corresponds to the inequality  
 $-12.5x + 25y - 100 < 0$  ?



**General Strategies:**

1. Read the question.
2. Highlight key words.
3. Identify the math topic.
4. Re-read the question.
5. Refer to your memory aid, as needed.
6. Look carefully at each choice shown (A, then B, then C and then D).
7. Eliminate options you know to be incorrect.
8. Solve/check each possible choice.
9. Select the choice that makes the most sense.

Do not leave a blank!  
 Make a choice!

**My Strategies:**

**Answer and Solution:**

$$-12.5x + 25y - 100 < 0$$

Test (0, 0) to see if the origin is in the solution set (and shaded).

$$\begin{aligned} -12.5(0) + 25(0) - 100 &< 0 \\ -100 &< 0 \\ \text{True} \end{aligned}$$

Therefore the answer must be D.

- A) Incorrect – shading not below the line
- B) Incorrect – solid line
- C) Incorrect – solid line
- D) **Correct – dashed line and shading below the line**

The answer is D.

**Specific Strategies:**

Since all the lines are the same (same initial value, same slope) there is no need to actually graph the given line.

What is different among the 4 choices?

- The shading is either above the line or below the line.
- The line is either dashed or solid.

Since the inequality sign doesn't have the bar underneath, you can eliminate choices B and C.

Now to decide if the shading is above or below

- Test a point and see if the inequality is true.

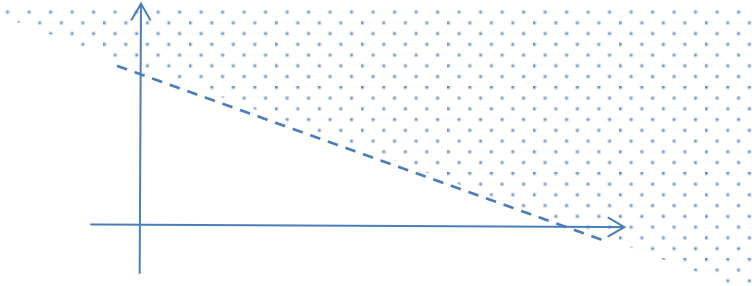
**Additional Resources:**

Visions Volume 1, Section 1.4, pp. 49-50

Khan Academy video: <http://www.khanacademy.org/math/algebra/linear-equations-and-inequalitie/graphing-linear-inequalities/v/graphing-inequalities>

**Question:**

Which rule represents the inequality graphed below?



- A)  $2y \leq 150x + 50$
- B)  $0 > -3y - x + 75$
- C)  $y \geq -\frac{1}{3}x + 25$
- D)  $y > 75x + 25$

**General Strategies:**

1. Read the question.
2. Highlight key words.
3. Identify the math topic.
4. Re-read the question.
5. Refer to your memory aid, as needed.
6. Look carefully at each choice shown (A, then B, then C and then D).
7. Eliminate options you know to be incorrect.
8. Solve/check each possible choice.
9. Select the choice that makes the most sense.

Do not leave a blank!  
Make a choice!

**My Strategies:**

**Answer and Solution:**

Test for B only since A, C and D are eliminated.

$$\begin{aligned} \text{Test } (0, 0) \text{ in} \quad & 0 > -3y - x + 75 \\ & y > -\frac{1}{3}x + 25 \\ & 0 > 0 + 25 \\ & 0 > 25 \\ & \text{is false.} \end{aligned}$$

Put all the rules into function form and compare:

A	$y \leq 75x + 25$	Not a dashed line
<b>B</b>	$y > -\frac{1}{3}x + 25$	<b>Correct – dashed line and negative slope</b>
C	$y \geq -\frac{1}{3}x + 25$	Not a dashed line
D	$y > 75x + 25$	Not a negative slope

The answer is B.

**Specific Strategies:**

Method 1:

Process of elimination. Before doing any work, you can eliminate answers A and C because the graph has a dashed line and therefore the rule must have either a less than or greater than sign (<, >).

You can also determine that the slope is negative since the line is sloping down. This eliminates D since that slope is positive.

**Additional Resources:**

Visions Volume 1, Section 1.4, pp. 49-50

Khan Academy video: <http://www.khanacademy.org/math/algebra/linear-equations-and-inequalitie/graphing-linear-inequalities/v/graphing-inequalities>

**Question:**

Consider the inequality  $4x - 2y < 8$  and point P (14, 24).

Is point P a solution to the inequality?

Yes, P is a solution to the inequality.

No, P is not a solution to the inequality.

**General Strategies:**

1. Read the question.
  2. Highlight key words.
  3. Identify the math topic.
  4. Re-read the question.
  5. Make a prediction about the answer- what will it look like? (an equation, a number, etc.).
  6. Refer to your memory aid, as needed.
  7. Solve.
  8. Ask yourself whether your answer makes sense.
  9. Write your answer.
- Do not leave a blank!

**My Strategies:**



**Answer and Solution:**

P (14, 24) and  $4x - 2y < 8$

$$4(14) - 2(24) < 8$$

$$56 - 48 < 8$$

$$8 < 8$$

Since 8 is not less than 8, the statement is false and P is not in the solution set.

Yes, P is a solution to the inequality.

No, P is not a solution to the inequality.

**Suggested Strategies:**

Check to see if that point, when plugged into the inequality, makes the statement true.

If it's true, it is a solution.

If it's false, it's not a solution.

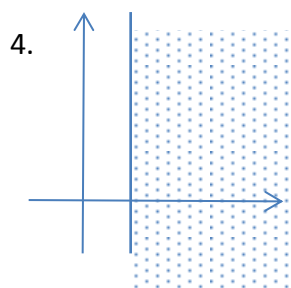
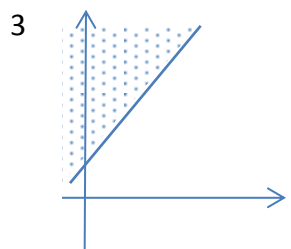
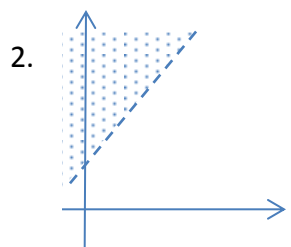
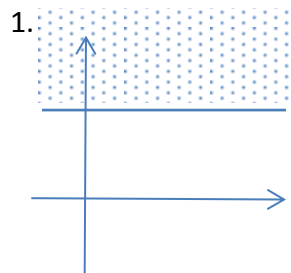
**Additional Resources:**

Visions Volume 1, Section 1.4, pp. 49-50

Khan Academy video: <http://www.khanacademy.org/math/algebra/linear-equations-and-inequalitie/graphing-linear-inequalities/v/graphing-inequalities>

**Question:**

Match the graphs to their corresponding rule:



A	$-3x + y - 5 \geq 0$
B	$-3x - 5 > -y$
C	$y - 5 \geq 0$
D	$0 \geq -x + 5$

Answers:

1. \_\_\_\_\_

2. \_\_\_\_\_

3. \_\_\_\_\_

4. \_\_\_\_\_

**General Strategies:**

1. Read the question.
  2. Highlight key words.
  3. Identify the math topic.
  4. Re-read the question.
  5. Make a prediction about the answer- what will it look like? (an equation, a number, etc.).
  6. Refer to your memory aid, as needed.
  7. Solve.
  8. Ask yourself whether your answer makes sense.
  9. Write your answer.
- Do not leave a blank!

**My Strategies:**

**Answer and Solution:**

A	$-3x + y - 5 \geq 0$	$y \geq 3x + 5$
B	$-3x - 5 > -y$	$y > 3x + 5$
C	$y - 5 \geq 0$	$y \geq 5$
D	$0 \geq -x + 5$	$x \geq 5$

**Answers:**

1.   C
2.   B
3.   A
4.   D

**Suggested Strategies:**

Put all of the inequalities into function form. Move  $y$  to the side where it will be positive (or the  $x$ , if there is no  $y$ .)

Once they are all in function form, look for information which is particular to only 1 graph.

- Rule B is the only dashed line ( $>$ ) so it must be graph 2.
- Rule A is the same as rule B except it is solid ( $\geq$ ) so it must be graph 3.
- Rule C has a constant slope so it must be graph 1.
- Rule D has an undefined slope (a vertical line) so it must be graph 4.

**Additional Resources:**

Visions Volume 1, Section 1.4, pp. 49-50

Khan Academy video: <http://www.khanacademy.org/math/algebra/linear-equations-and-inequalitie/graphing-linear-inequalities/v/graphing-inequalities>

**Question:**

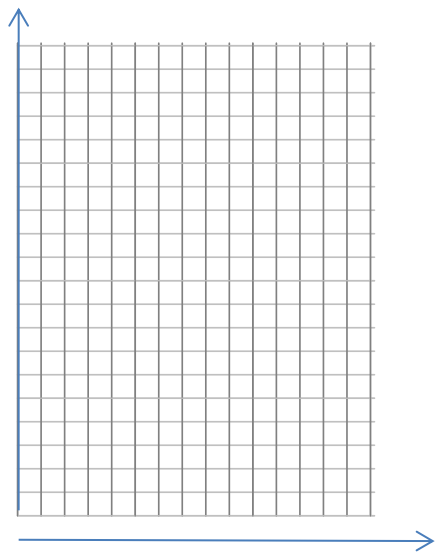
Juanita studies the layout of the IMAX theater by her house. Considering the angle of the seats, the width of the screen and the sound system, she comes up with a theory of where the best seats are.

She represents her theory as three inequalities on a Cartesian plane. The Cartesian plane represents the theater, and each of the intersections (lattice points) represents one seat.

Juanita's inequalities:

$$\begin{aligned} 4x &< -3y + 72 \\ y &< \frac{4}{3}x + 6 \\ y &\geq 10 \end{aligned}$$

The area where all three inequalities overlap contains the best seats.



How many of the seats can be considered the "best seats"?

**General Strategies:**

1. Read the problem.
2. Highlight key words.
3. Identify the math topics.
4. Re-read the problem.
5. Define your steps (your game plan) – this is criteria 3.
6. Refer to your memory aid, as needed.
7. Solve.
8. If you get stuck on a calculation, pick a number and keep going.
9. Ask yourself whether your answer makes sense.
10. Write your answer statement.

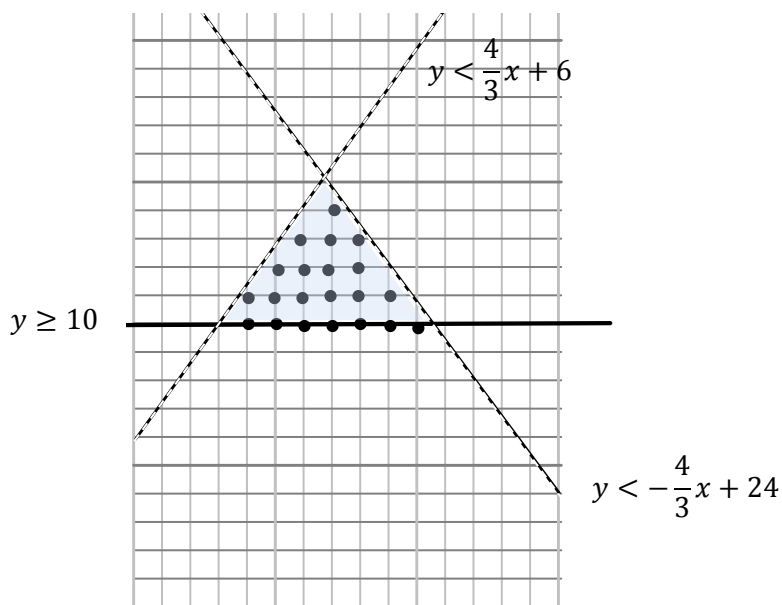
Show any or all your work! Do not leave a blank page!

**My Strategies:**

**Answer and Solution:**

Juanita's inequalities:

1.  $4x < -3y + 72$   
 $3y < -4x + 72$   
 $y < -\frac{4}{3}x + 24$
2.  $y < \frac{4}{3}x + 6$
3.  $y \geq 10$



There are 21 seats in the region of "the best seats".

**Suggested Strategies:**

Graph and shade each of the inequalities on the same Cartesian plane. Identify the area which is shaded by all three inequalities (It's a triangle).

Count the intersection points that are in the triangle. Include the ones that are on the solid line, but don't include the ones on the dashed line. (e.g. (3, 10) touches both a solid line and a dashed line and should be excluded.)

Remember that any point can be checked by putting the coordinates into all the inequalities to see if it is 'true'. If it isn't 'true' for all 3 inequalities it is not one of the "best seats."

**Additional Resources:**

Visions Volume 1, Section 1.4, pp. 49-50

Khan Academy video: <http://www.khanacademy.org/math/algebra/linear-equations-and-inequalitie/graphing-linear-inequalities/v/graphing-inequalities>



## **2.1 Diagrams and Statistics (Dispersion, Deviation, Stem and Leaf...)**

**Question:**

What is the mean deviation of the following set of data?

21 21 21 23 23 23

- A) 0
- B) 1
- C) 2
- D) 6

**General Strategies:**

1. Read the question.
2. Highlight key words.
3. Identify the math topic.
4. Re-read the question.
5. Refer to your memory aid, as needed.
6. Solve the problem without looking at choices shown (A, B, C and D).
7. Look at all the choices.
8. Match your answer to the appropriate choice.

Do not leave a blank!  
Make a choice!

**My Strategies:**



**Answer and Solution:**

1. Mean =  $(21+21+21+23+23+23) \div 6$   
Mean = 22
  2.  $|21-22| = |-1| = 1$   
 $|21-22| = |-1| = 1$   
 $|21-22| = |-1| = 1$   
 $|23-22| = 1$   
 $|23-22| = 1$   
 $|23-22| = 1$
  3. Mean deviation =  $(1+1+1+1+1+1) \div 6$   
Mean deviation = 1
- A) 0 – if you didn't take the absolute value of the differences
- B) **1 – correct**
- C) 2 – unlikely, but just in case you take the difference between the two repeated values
- D) 6 – if you forget to divide by 6

The answer is B.

**Specific Strategies:**

1. Calculate the mean of the set of data.
2. Subtract the mean from each value in the set of data and determine its absolute value. (Remember absolute values can't be negative.)
3. Calculate the mean of the deviations.

**Additional Resources:**

Visions Volume 1, p. 81

[http://www.wikihow.com/Calculate-Mean-Deviation-About-Mean-\(for-Ungrouped-Data\)](http://www.wikihow.com/Calculate-Mean-Deviation-About-Mean-(for-Ungrouped-Data))

**Question:**

Which of the following statements is/are true concerning statistical measures?

- I. The mean, median and range are measures of central tendency.
  - II. Percentile rank is a measure of dispersion.
  - III. The mean deviation and range are measures of dispersion.
  - IV. The mean deviation is a measure of position.
- A) I only
- B) III only
- C) II and III only
- D) II and IV only

**General Strategies:**

1. Read the question.
2. Highlight key words.
3. Identify the math topic.
4. Re-read the question.
5. Refer to your memory aid, as needed.
6. Solve the problem without looking at choices shown (A, B, C and D).
7. Look at all the choices.
8. Match your answer to the appropriate choice.

Do not leave a blank!  
Make a choice!

**My Strategies:**

**Answer and Solution:**

- I. The mean, median, and range are measures of central tendency. False – range is a measure of dispersion.
- II. Percentile rank is a measure of dispersion. False – percentile is a measure of position.
- III. **The mean deviation and range are measures of dispersion. True.**
- IV. The mean deviation is a measure of position. False – mean deviation is a measure of dispersion.

**Suggested Strategies:**

It is important to remember your vocabulary. Instead of blindly calculating mean, range etc. Try to think about why you are doing them and what the result represents. This goes for any stats question.

## Recall:

- The measures of central tendency are mean, median, and mode.
- The measures of position are percentile rank
- Range and mean deviation are measures of dispersion.

The answer is B.

**Additional Resources:**

Visions Volume 2, pp. 81-82

Khan Academy video: <https://www.khanacademy.org/math/cc-seventh-grade-math/cc-7th-probability-statistics/cc-7th-central-tendency/v/statistics-intro--mean--median-and-mode>

**Question:**

Consider the stem-leaf plot below showing the number of sit-ups students do in 60 seconds.

	Number of Sit-ups
2	0 1 1 2 2 8 9
3	2 2 3 4 5 6 6 8 9
4	1 1 2 3 4 4 4 5 6 7 8
5	0 1 1 2 5 6 6 7 8 8 8
6	2 4 6 6

How many sit-ups did a student do if he is ranked in the 70<sup>th</sup> percentile?

- A) 35
- B) 36
- C) 51
- D) 52

**General Strategies:**

1. Read the question.
2. Highlight key words.
3. Identify the math topic.
4. Re-read the question.
5. Refer to your memory aid, as needed.
6. Solve the problem without looking at choices shown (A, B, C and D).
7. Look at all the choices.
8. Match your answer to the appropriate choice.

Do not leave a blank!  
Make a choice!

**My Strategies:**

**Answer and Solution:**

1. There are 42 values in the stem and leaf plot
2. Apply the formula:  $\frac{\textit{percentile}}{100} \times \textit{number of values}$

$$\frac{70}{100} \times 42$$

$$0.70 \times 42$$

$$29.4$$

3. Always round down the result (round to 29).
4. The answer represents the POSITION from the bottom (lowest result) of the data value you wish to find.

	Number of sit-ups
2	0 1 1 2 2 8 9
3	2 2 3 4 5 6 6 8 9
4	1 1 2 3 4 4 4 5 6 7 8
5	0 1 2 2 5 6 6 7 8 8 8
6	2 4 6 6

29<sup>th</sup> value is 51

- A) This is the result when you make both mistakes as described in B) and D).
- B) This is the result when you count from the wrong end.
- C) This is correct.**
- D) This is the result when you round up instead of down – it’s the 30<sup>th</sup> position.

The answer is C.

**Specific Strategies:**

Read the stem and leaf plot correctly.

Do the calculations completely before selecting your answer.

The percentile is given which means you need to work backwards to find the data value.

1. Find the total amount of values
2. Find the position from the bottom by applying the formula
3. Locate that position according to value found in step 3.

**Additional Resources:**

Khan Academy Video: [https://www.khanacademy.org/math/arithmetric/applying-math-reasoning-topic/reading\\_data/e/reading\\_stem\\_and\\_leaf\\_plots](https://www.khanacademy.org/math/arithmetric/applying-math-reasoning-topic/reading_data/e/reading_stem_and_leaf_plots)  
<http://www.purplemath.com/modules/stemleaf2.htm>

**Question:**

At a large company, a survey was conducted to see how fast employees can type. The company has 305 employees. The partial list below shows the speed, in words per minute, achieved by the employees:

**30, 32, 32, ... 49, 50, 50, 50, 52, 53, 53, ... 89, 90, 93, 99**

146 employees    3 employees    156 employees

What is the percentile rank for an employee who types 50 words per minute?

- A) 47
- B) 48
- C) 49
- D) 50

**General Strategies:**

1. Read the question.
2. Highlight key words.
3. Identify the math topic.
4. Re-read the question.
5. Refer to your memory aid, as needed.
6. Solve the problem without looking at choices shown (A, B, C and D).
7. Look at all the choices.
8. Match your answer to the appropriate choice.

Do not leave a blank!  
Make a choice!

**My Strategies:**

**Answer and Solution:**

Answer using the formula:

$$\frac{\text{number of data values below } x + \frac{\text{number of data values equal to } x}{2}}{\text{total number of data values}} \times 100$$

$$\frac{146 + \frac{3}{2}}{305} \times 100 = \frac{146 + 1.5}{305} \times 100 = \frac{147.5}{305} \times 100$$

$$\approx 48.36 \text{ round up to } \mathbf{49}$$

- A) 47 – if you don't take into account the 3 values at 50
- B) 48 – if you round down or don't take into account the 3 values at 50
- C) 49 – correct**
- D) 50 – if you just take the value itself

The answer is C.

**Suggested Strategies:**

The question is asking for percentile, so you need the formula which gives you the percentile of a data value.

Be careful:

Do not use the formula for finding a data value when the percentile is given!

Remember to round to the next whole number (always UP!)

**Additional Resources:**

Visions Volume 1, Section 2.1, pp. 76-88

**Question:**

Consider the following set of data:

41 17 25 9 20 12 11 21 20

What is the mean deviation for the set of data?

The mean deviation for the set of data is \_\_\_\_\_.

**General Strategies:**

1. Read the question.
  2. Highlight key words.
  3. Identify the math topic.
  4. Re-read the question.
  5. Make a prediction about the answer- what will it look like? (an equation, a number, etc.).
  6. Refer to your memory aid, as needed.
  7. Solve.
  8. Ask yourself whether your answer makes sense.
  9. Write your answer.
- Do not leave a blank!


**My Strategies:**



**Answer and Solution:**

It is suggested to organize your work in a table.

- The mean of the set of data is 19.55.
- See the third column of the table.



Value	Mean	Difference of value from mean
41	19.55	21.45
17	19.55	2.55
25	19.55	5.45
9	19.55	10.55
20	19.55	0.45
12	19.55	7.55
11	19.55	8.55
21	19.55	1.45
20	19.55	0.45

Total: 58.45

- The mean deviation is:

$$\text{Mean deviation} = \frac{58.45}{9} = 6.49$$

The mean deviation for the set of data is 6.49.

**Specific Strategies:**

- Calculate the mean of the set of data
- Subtract the mean from each value in the set of data
- Calculate the mean of the deviations

Remember to take the absolute value of each deviation  
NO NEGATIVE VALUES!

**Additional Resources:**

Visions Volume 1, p. 81

<http://www.mathsisfun.com/data/mean-deviation.html>

QUESTION 30

2.1-SA-B

**Question:**

The table below shows the finishing times for the 137 runners participating in a 5 km race:

18:48	26:36	29:22	31:34	35:08	38:04	48:58
20:01	26:37	29:29	31:55	35:09	38:45	48:58
21:19	26:43	29:29	32:13	35:09	38:59	49:50
21:55	26:48	29:30	32:26	35:35	39:21	49:51
23:36	26:54	29:30	32:28	35:39	39:22	50:01
23:36	27:20	29:31	32:28	35:45	39:38	53:40
24:15	27:38	29:49	32:36	36:11	40:52	56:19
24:29	27:50	29:56	32:50	36:11	41:07	56:20
24:34	27:50	30:02	32:56	36:12	41:35	57:06
24:34	27:56	30:03	32:57	36:24	41:35	59:12
24:35	28:32	30:08	33:07	36:25	44:44	59:14
25:01	28:42	30:28	33:09	36:25	44:45	59:18
25:04	<b>28:45</b>	30:31	33:14	36:27	46:01	1:00:55
25:08	<b>28:45</b>	30:31	33:30	37:21	46:15	1:01:05
25:08	28:59	30:34	33:39	37:21	46:22	1:03:39
25:44	29:02	30:39	33:46	37:25	46:24	1:03:42
25:58	29:04	31:07	33:46	37:43	47:05	1:03:46
26:19	29:13	31:25	33:46	37:54	47:19	
26:24	29:17	31:27	34:22	37:58	47:19	
26:31	29:17	31:29	34:43	38:03	48:11	

- A) What is the percentile rank of a runner with a finishing time of 28 minutes 45 seconds?
- B) What is the finishing time of the runner who ranked in the 60<sup>th</sup> percentile?

The percentile rank of the runner finishing with 28:45 is \_\_\_\_\_.

The finishing time of the runner ranked in the 60<sup>th</sup> percentile is \_\_\_\_\_.

**General Strategies:**

1. Read the question.
2. Highlight key words.
3. Identify the math topic.
4. Re-read the question.
5. Make a prediction about the answer- what will it look like? (an equation, a number, etc.).
6. Refer to your memory aid, as needed.
7. Solve.
8. Ask yourself whether your answer makes sense.
9. Write your answer.

Do not leave a blank!

**My Strategies:**

**Answer and Solution:****Solution A**

Percentile rank=

$$\frac{\text{total number of values below} + \frac{\text{total number of values equal to}}{2}}{\text{total number of values}} \times 100$$

$$\frac{103 + \frac{2}{2}}{137} \times 100 = 75.9$$

Round up to the next integer: 76<sup>th</sup>**Solution B**

Percentile rank =

$$\frac{\text{percentile rank}}{100} \times \# \text{ of values} = \text{position}$$

$$\frac{60}{100} \times 137 = 82.2 \text{ or } 82 \text{ from the bottom}$$

or

$$137 - 82 = 55 \text{ runners finished ahead of him}$$

He is the 56<sup>th</sup> runner from the top/82<sup>nd</sup> from the bottom with a time of 30:39.

Note: Your answer might be different if you use a different percentile formula.

**A) The percentile rank of a runner finishing with 28:45 is the 76<sup>th</sup>.**

**B) The finishing time of a runner ranked in the 60<sup>th</sup> percentile is 30:39.**

**Suggested Strategies:**

Is the question asking for the percentile rank or the value at a specific position?

This will help determine which equation to use.

Remember a longer time is located at the bottom of the list.

1. First make sense of the table and notice that the numbers increase as you go down and across. There are 20 rows – use that fact as you count positions.
2. You'll notice there are two runners with a time of 28:45 and that they were faster than 103 of the other runners.
3. Apply the formula for finding percentile.
4. Round up to the next integer
5. Reverse the procedure to find the time for the 60<sup>th</sup> percentile.

**Additional Resources:**

Visions Volume 1, p. 81-82

**Question:**

The 20 best swimmers from across the country are trying out for the national swim team. To earn a spot on the team, a swimmer must meet both the following criteria:

**Criteria 1**

The swimmer must rank better than the 60<sup>th</sup> percentile.

**Criteria 2**

The swimmer must have a “personal best time” (PBT) that is less than or equal to 20 seconds *minus* the mean deviation (MD) of the group.

$$PBT \leq 20 - MD$$

**“Personal Best Times”  
(in seconds)**

18.56	19.25	19.92	20.2
18.7	19.26	19.92	20.4
18.9	19.8	19.94	20.8
18.95	19.85	19.96	20.8
19.2	19.9	19.99	21.1

The mean of this distribution is 19.77 seconds.

How many of the 20 swimmers will earn a spot on the National team?

**General Strategies:**

1. Read the problem.
2. Highlight key words.
3. Identify the math topics.
4. Re-read the problem.
5. Define your steps (your game plan) – this is criteria 3.
6. Refer to your memory aid, as needed.
7. Solve.
8. If you get stuck on a calculation, pick a number and keep going.
9. Ask yourself whether your answer makes sense.
10. Write your answer statement.

Show any or all your work! Do not leave a blank page!

**My Strategies:**

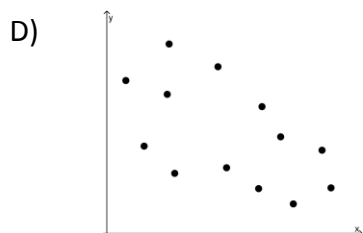
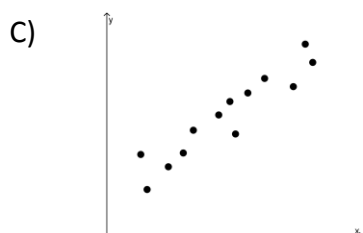
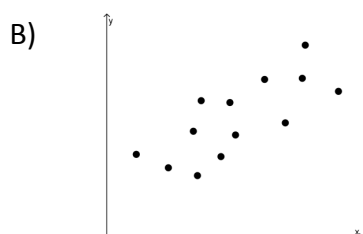
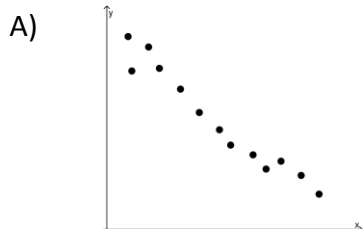
<p><b>Answer and Solution:</b></p> <p><b>Criteria 1:</b></p> <p>Swimmers that meet qualification 1:</p> $\frac{60}{100}(20) = 12$ <p>This means 12 swimmers are at or below the 60<sup>th</sup> percentile.</p> <p>If 12 of the 20 swimmers are at or below the 60<sup>th</sup> percentile, then 8 are above it.</p> <p><b>Criteria 2:</b></p> <p>Mean of the distribution: 19.77 (sum of all values ÷ 20)</p> <p>Mean deviation of the distribution: 0.557 (sum of all mean deviations ÷ 20)</p> $\begin{aligned}  18.56 - 19.77  &= 1.21 \\  18.7 - 19.77  &= 1.07 \\ &\vdots \\  21.1 - 19.77  &= 1.33 \end{aligned}$ <p style="text-align: center;">Sum of deviations = 11.14 11.14 ÷ 20 = 0.557</p> <p>Swimmers that meet qualification 2.</p> <p>PBT ≤ 20 – MD PBT ≤ 20 – 0.557 PBT ≤ 19.443</p> <p>They are: 18.56, 18.7, 18.9, 18.95, 19.2, 19.25, 19.26</p> <p><b>Seven (7) swimmers will earn a spot on the National team.</b></p>	<p><b>Specific Strategies:</b></p> <p>Remember that the lowest scores are the best scores. Therefore, the values in this distribution are given in order of best to worst, not from worst to best (which is what we usually see). This makes finding the percentile a little bit tricky.</p> <p>Make sure that your answer takes into account the requirement to meet or exceed both of the evaluations that are used to select team members.</p>
<p><b>Additional Resources:</b></p> <p>Visions Volume 1, pp. 81-82</p>	



# 2.2 Qualitative Interpretation of Correlation

**Question:**

Which of the following scatterplots shows the strongest linear correlation?



**General Strategies:**

1. Read the question.
2. Highlight key words.
3. Identify the math topic.
4. Re-read the question.
5. Refer to your memory aid, as needed.
6. Look carefully at each choice shown (A, then B, then C and then D).
7. Eliminate options you know to be incorrect.
8. Solve/check each possible choice.
9. Select the choice that makes the most sense.

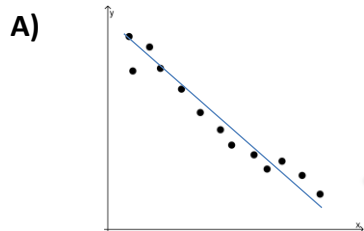
Do not leave a blank!  
Make a choice!

**My Strategies:**

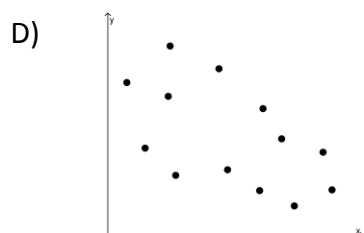
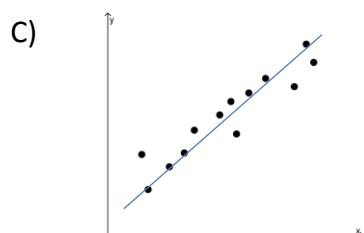
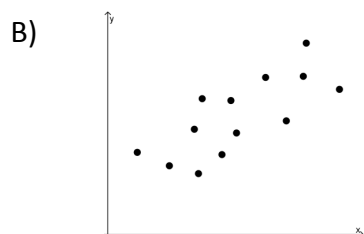


**Answer and Solution:**

Answers B) and D) can be ruled out since the points are quite far apart.



← The points are closest to forming a straight line but strongest in A)



The answer is A.

**Specific Strategies:**

The scatterplot that shows a distribution of points closest to forming a straight line has the strongest correlation.

The strength of a correlation is not related to its direction (or sign).

**Additional Resources:**

Visions Volume 1, Section 2.2, pp. 93-95

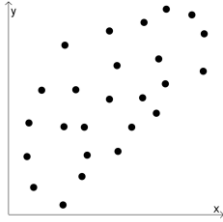
Explore Learning Gizmos, <http://www.explorelarning.com/> look up:

- Correlation
- Trends in Scatter Plots
- Scatter Plots-Activity

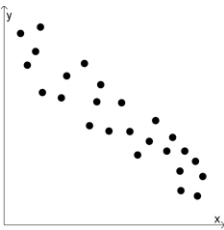
**Question:**

Which of the following scatterplots best shows a zero correlation?

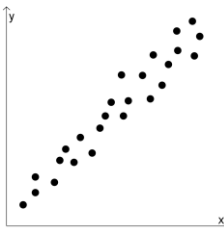
A)



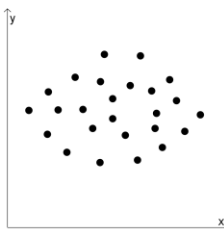
B)



C)



D)

**General Strategies:**

1. Read the question.
2. Highlight key words.
3. Identify the math topic.
4. Re-read the question.
5. Refer to your memory aid, as needed.
6. Look carefully at each choice shown (A, then B, then C and then D).
7. Eliminate options you know to be incorrect.
8. Solve/check each possible choice.
9. Select the choice that makes the most sense.

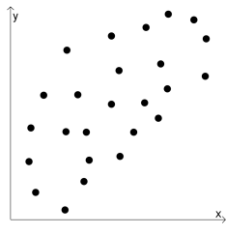
Do not leave a blank!

Make a choice!

**My Strategies:**

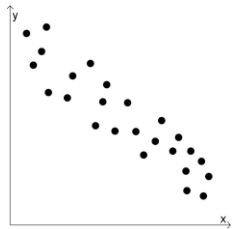
**Answer and Solution:**

A)

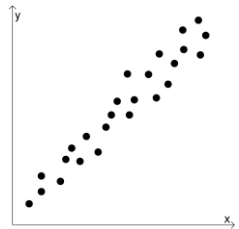


a slight positive line, so not 0

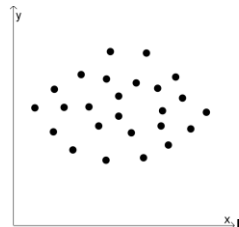
B)

a fairly significant clustering  
around a negative line

C)

a rather tight clustering around  
a positive line

D)



no tendency towards a line

**Suggested Strategies:**

The scatterplot that shows a distribution of points with no clear direction and furthest from forming a straight line indicates a zero correlation.

**The answer is D.****Additional Resources:**

Visions Volume 1, Section 2.2, pp. 93-95

Explore Learning Gizmos, <http://www.explorellearning.com/> look up:

- Correlation
- Trends in Scatter Plots
- Scatter Plots-Activity

**Question:**

At a recent school event, students were asked to sit facing the stage. The following table shows the distribution of students according to their ages and the distance from the stage.

AGE (years) \ DISTANCE (m)	[2, 4[	[4, 6[	[6, 8[	[8, 10[	[10, 12[
[10, 11[	3	3	3	3	3
[11, 12[	3	3	3	3	3
[12, 13[	3	3	3	3	3
[13, 14[	3	3	3	3	3
[14, 15[	3	3	3	3	3

Which of the following best describes the linear correlation between the age of the students and the distance from each student to the stage?

- A) The correlation is positive.
- B) The correlation is negative.
- C) The correlation is perfect.
- D) The correlation is zero.

**General Strategies:**

1. Read the question.
2. Highlight key words.
3. Identify the math topic.
4. Re-read the question.
5. Refer to your memory aid, as needed.
6. Look carefully at each choice shown (A, then B, then C and then D).
7. Eliminate options you know to be incorrect.
8. Solve/check each possible choice.
9. Select the choice that makes the most sense.

Do not leave a blank!  
Make a choice!

**My Strategies:**

**Answer and Solution:**

DISTANCE (m) AGE (years)	[2, 4[	[4, 6[	[6, 8[	[8, 10[	[10, 12[
[10, 11[	3	3	3	3	3
[11, 12[	3	3	3	3	3
[12, 13[	3	3	3	3	3
[13, 14[	3	3	3	3	3
[14, 15[	3	3	3	3	3

In a table, the closer the data is to the diagonal, the stronger the correlation.

All values are 3 so data is not clustered on the diagonal.

In this case, the data is evenly spread out, thus indicating no correlation.

- A) The correlation is positive – false: data is not clustered around a diagonal from top left to bottom right.
- B) The correlation is negative – false: data is not clustered around a diagonal from bottom left to top right.
- C) The correlation is perfect – false: given an age, there is no way to predict the distance from the stage.
- D) The correlation is zero – true:** given an age, there is no way to predict the distance from the stage.

**The answer is D.**

**Specific Strategies:**

The closer the data is to the diagonal, the stronger the correlation.

A correlation may be zero, weak, moderate, strong or perfect!

**Additional Resources:**

Visions Volume 1, Section 2.2, pp. 93-94

**Question:**

Consider the following table showing a two-variable distribution. Indicate the strength and direction of correlation.

$x \backslash y$	[0,1[	[1,2[	[2,3[	[3,4[	[4,5[
1	2	0	0	0	0
2	0	2	0	0	0
3	0	3	2	2	0
4	0	0	0	5	2
5	0	0	0	1	1

**General Strategies:**

1. Read the question.
  2. Highlight key words.
  3. Identify the math topic.
  4. Re-read the question.
  5. Make a prediction about the answer- what will it look like? (an equation, a number, etc.).
  6. Refer to your memory aid, as needed.
  7. Solve.
  8. Ask yourself whether your answer makes sense.
  9. Write your answer.
- Do not leave a blank!

**My Strategies:**

Strength

- Weak
- Strong

Direction

- Positive
- Negative

**Answer and Solution:**

$x \backslash y$	$[0,1[$	$[1,2[$	$[2,3[$	$[3,4[$	$[4,5[$
1	2	0	0	0	0
2	0	2	0	0	0
3	0	3	2	2	0
4	0	0	0	5	2
5	0	0	0	1	1

**Suggested Strategies:**

- In a table, the closer the data is to the diagonal, the stronger the correlation.
- If the diagonal slopes downward, then the correlation is positive – because as  $x$  increases, so does  $y$ .

**The answer is:**

Strength

Weak

Strong

Direction

Positive

Negative

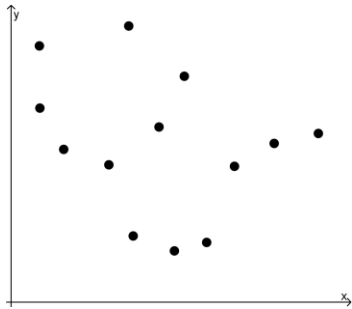
**Additional Resources:**

Visions Volume 1, Section 2.2, pp. 93-94

**Question:**

Consider the following scatterplot.

- A) Is the correlation weak or strong?  
 B) Is the direction positive or negative?

**General Strategies:**

1. Read the question.
  2. Highlight key words.
  3. Identify the math topic.
  4. Re-read the question.
  5. Make a prediction about the answer- what will it look like? (an equation, a number, etc.).
  6. Refer to your memory aid, as needed.
  7. Solve.
  8. Ask yourself whether your answer makes sense.
  9. Write your answer.
- Do not leave a blank!

**My Strategies:**Strength

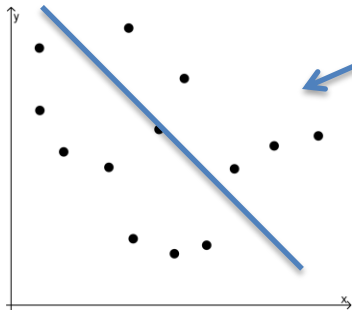
- Weak  
 Strong

Direction

- Positive  
 Negative



**Answer and Solution:**



- The points appear to form a descending line (negative direction)
- The points are very scattered far apart (weak correlation)

**Specific Strategies:**

Look at how far or close the points are relative to each other

- The closer the points are to forming a straight line, the stronger the correlation is
- A positive slope means a positive correlation
- A negative slope means a negative correlation

**The answer is:**

Strength

Weak

Strong

Direction

Positive

Negative

**Additional Resources:**

Visions Volume 1, Section 2.2, pp. 93-95

Explore Learning Gizmos, <http://www.explorellearning.com/> look up:

- Correlation
- Trends in Scatter Plots
- Scatter Plots-Activity



## **2.3 Quantitative Interpretation of Correlation**

**Question:**

The table below shows the linear correlation coefficient between the two variables of four different statistical distributions.

Distribution	Linear Correlation Coefficient
1	-0.81
2	0.39
3	-0.27
4	0.74

Which of the following presents these distributions, in order, from weakest to strongest linear correlation?

- A) 1, 4, 2, 3
- B) 3, 2, 4, 1
- C) 1, 3, 2, 4
- D) 4, 2, 3, 1

**General Strategies:**

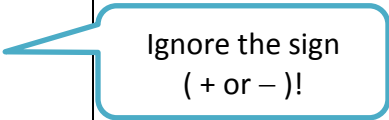
1. Read the question.
2. Highlight key words.
3. Identify the math topic.
4. Re-read the question.
5. Refer to your memory aid, as needed.
6. Look carefully at each choice shown (A, then B, then C and then D).
7. Eliminate options you know to be incorrect.
8. Solve/check each possible choice.
9. Select the choice that makes the most sense.

Do not leave a blank!  
Make a choice!

**My Strategies:**

**Answer and Solution:**

- A)  $-0.81, 0.74, 0.39, -0.27$   
is from strongest to weakest
- B)  $-0.27, 0.39, 0.74, -0.81$   
Correct**
- C)  $-0.81, -0.27, 0.39, 0.74$   
is from smallest number to greatest (keeps the sign)
- D)  $0.74, 0.39, -0.27, -0.81$   
is from largest number to smallest number (keeps the sign)

**Specific Strategies:**

Ignore the sign  
( + or - )!

- Order the coefficients (without the sign) in increasing order.
- The coefficient closest to 0 is the weakest.
- The coefficient closest to 1 is the strongest.

The answer is B.

**Additional Resources:**

Visions Volume 1, Section 2.3, p. 110

Explore Learning Gizmos, <http://www.explorelearning.com/> look up:

- Correlation
- Trends in Scatter Plots
- Scatter Plots-Activity

**Question:**

Which of the following correlation coefficient shows the weakest correlation?

- A)  $-0.75$
- B)  $-0.45$
- C)  $0.16$
- D)  $0.83$

**General Strategies:**

1. Read the question.
2. Highlight key words.
3. Identify the math topic.
4. Re-read the question.
5. Refer to your memory aid, as needed.
6. Look carefully at each choice shown (A, then B, then C and then D).
7. Eliminate options you know to be incorrect.
8. Solve/check each possible choice.
9. Select the choice that makes the most sense.

Do not leave a blank!  
Make a choice!

**My Strategies:**

**Answer and Solution:**

- A)  $-0.75$  – don't mistake the smallest number with the weakest correlation
- B)  $-0.45$  – is further from 0 than 0.16
- C)  $0.16$  – correct**
- D)  $0.83$  – this is the strongest correlation shown

**Specific Strategies:**

Ignore the + or – signs!

The coefficient closest to 0 is the weakest

The answer is C.

**Additional Resources:**

Visions Volume 1, Section, 2.3 p. 110

Explore Learning Gizmos, <http://www.explorellearning.com/> look up:

- Correlation
- Trends in Scatter Plots
- Scatter Plots-Activity

**Question:**

Which of the following correlation coefficients shows a perfect correlation?

- A)  $-1.00$
- B)  $-0.10$
- C)  $0.00$
- D)  $0.99$

**General Strategies:**

1. Read the question.
2. Highlight key words.
3. Identify the math topic.
4. Re-read the question.
5. Refer to your memory aid, as needed.
6. Look carefully at each choice shown (A, then B, then C and then D).
7. Eliminate options you know to be incorrect.
8. Solve/check each possible choice.
9. Select the choice that makes the most sense.

Do not leave a blank!  
Make a choice!

**My Strategies:**



**Answer and Solution:**

- A) **-1.00 – correct (perfect correlation can be positive or negative)**
- B) -0.10 – in case you misplace the decimal
- C) 0.00 – in case you mix up “perfect” and “no correlation”
- D) 0.99 – really close, but not perfect.

**Specific Strategies:**

Ignore the + or – signs!

A perfect correlation is equal to 1 or -1

**The answer is A.**

**Additional Resources:**

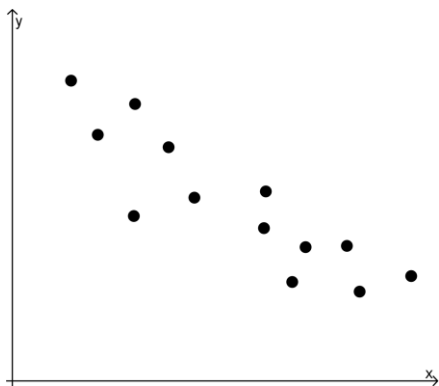
Visions Volume 1, Section 2.3, p. 110

Explore Learning Gizmos, <http://www.explorellearning.com/> look up:

- Correlation
- Trends in Scatter Plots
- Scatter Plots-Activity

**Question:**

Consider the following scatterplot.



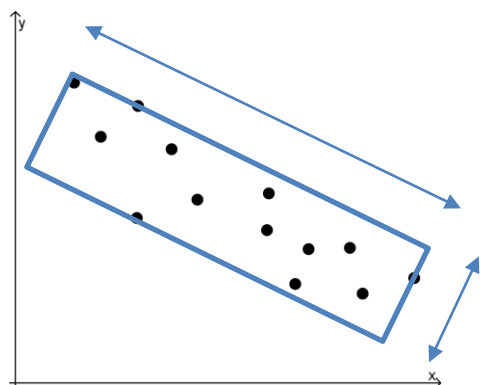
What is the linear correlation coefficient?

**General Strategies:**

1. Read the question.
  2. Highlight key words.
  3. Identify the math topic.
  4. Re-read the question.
  5. Make a prediction about the answer- what will it look like? (an equation, a number, etc.).
  6. Refer to your memory aid, as needed.
  7. Solve.
  8. Ask yourself whether your answer makes sense.
  9. Write your answer.
- Do not leave a blank!

**My Strategies:**

The linear correlation coefficient is \_\_\_\_\_.

**Answer and Solution:**

$$r = \pm \left( 1 - \frac{\text{short side}}{\text{long side}} \right)$$

$$r = - \left( 1 - \frac{1.4 \text{ cm}}{5.3 \text{ cm}} \right)$$

$$r = -(0.74)$$

Since the slope is negative, the correlation coefficient should also be negative

**The linear correlation coefficient is  $-0.74$ .**

*(Because of slight differences in measurement, your answer could be anywhere between  $-0.82$  and  $-0.66$ .)*

**Specific Strategies:**

1. Encase the points within the tightest rectangle possible.
2. Measure the dimensions of the rectangle.
3. Estimate the linear correlation coefficient using the formula:

$$r = \pm \left( 1 - \frac{\text{short side}}{\text{long side}} \right)$$

**Additional Resources:**

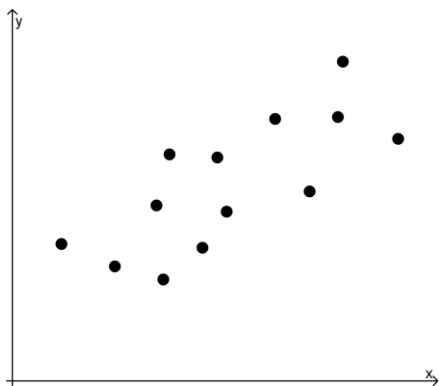
Visions Volume 1, Section 2.3, p. 110

Explore Learning Gizmos, <http://www.explorelarning.com/> look up:

- Correlation
- Trends in Scatter Plots
- Scatter Plots-Activity

**Question:**

Consider the following scatterplot.



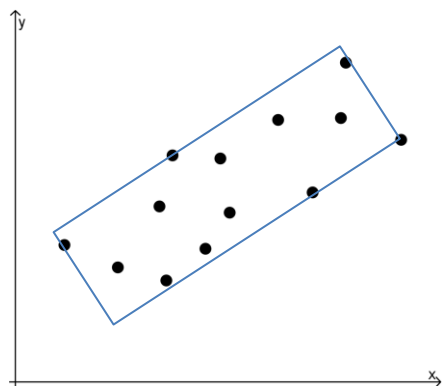
Estimate the linear correlation coefficient.

The linear correlation coefficient is \_\_\_\_\_.

**General Strategies:**

1. Read the question.
  2. Highlight key words.
  3. Identify the math topic.
  4. Re-read the question.
  5. Make a prediction about the answer—what will it look like? (an equation, a number, etc.).
  6. Refer to your memory aid, as needed.
  7. Solve.
  8. Ask yourself whether your answer makes sense.
  9. Write your answer.
- Do not leave a blank!

**My Strategies:**

**Answer and Solution:**

$$r = \pm \left( 1 - \frac{\text{short side}}{\text{long side}} \right)$$

$$r = + \left( 1 - \frac{1.5 \text{ cm}}{4.5 \text{ cm}} \right)$$

$$r = +(0.67)$$

Since the slope is positive, the correlation coefficient should also be positive.

**The linear correlation coefficient is 0.67.**

*(Because of slight differences in measurement, your answer could be anywhere between 0.59 and 0.75)*

**Specific Strategies:**

Encase the points with a rectangle. Measure the dimensions of the rectangle. Estimate the linear correlation coefficient using the formula:

$$r = \pm \left( 1 - \frac{\text{short side}}{\text{long side}} \right)$$

**Additional Resources:**

Visions Volume 1, Section 2.3, p. 110

Explore Learning Gizmos, <http://www.explorellearning.com/> look up:

- Correlation
- Trends in Scatter Plots
- Scatter Plots-Activity

**Question:**

The Granby Zoo feeds its elephants daily. The chart below shows the weight of several elephants and the weight of the food they are given every day.

Weight of elephant (kg)	Weight of food (kg)
1250	58
1300	63
1320	66
1382	69
1400	67
1460	63
1480	70
1492	76

How much food would an elephant weighing 1600 kg be given? Round your answer to the nearest tenth of a kilogram.

A 1600kg elephant would be given \_\_\_\_\_ kg of food.

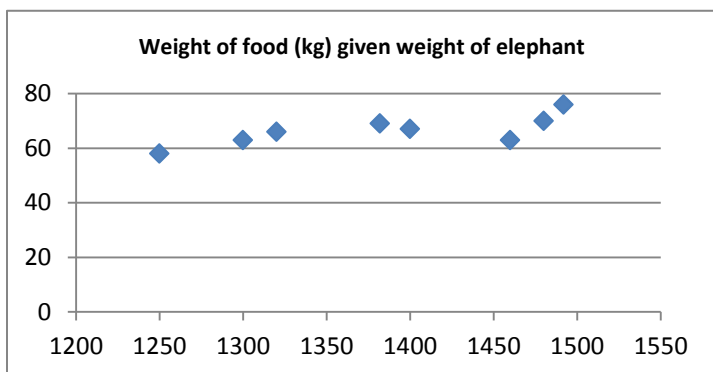
**General Strategies:**

1. Read the problem.
2. Highlight key words.
3. Identify the math topics.
4. Re-read the problem.
5. Define your steps (your game plan) – this is criteria 3.
6. Refer to your memory aid, as needed.
7. Solve.
8. If you get stuck on a calculation, pick a number and keep going.
9. Ask yourself whether your answer makes sense.
10. Write your answer statement.

Show any or all your work! Do not leave a blank page!

**My Strategies:**

**Answer and Solution:**



Perform a linear regression using the data in the table.

Mayer line method:

Let  $x$  be the weight of an elephant, and  $y$  be the weight of the food

Split data in half:

$x$	1250	1300	1320	1382
$y$	58	63	66	69

	1400	1460	1480	1492
	67	63	70	76

$$\text{average } x \text{ first half} = \frac{1250+1300+1320+1382}{4} = 1313$$

$$\text{average } x \text{ second half} = \frac{1400+1460+1480+1492}{4} = 1458$$

$$\text{average } y \text{ first half} = \frac{58+63+66+69}{4} = 64$$

$$\text{average } y \text{ second half} = \frac{67+63+70+76}{4} = 69$$

Point 1 (1313, 64)

Point 2 (1458, 69)

$$a = \frac{69 - 64}{1458 - 1313} = 0.0344$$

$$b = 64 - (0.0344)(1313) = 18.8328$$

Linear regression rule:  $y = 0.0344x + 18.8328$

Find the value of  $y$  when  $x = 1600$

$$y = 0.0344(1600) + 18.8328 = 73.8728$$

**A 1600 kg elephant would be given 73.9 kg of food. Your answer might vary.**

**Suggested Strategies:**

This is a case where we use data to come up with a rule in order to predict, or in this case extrapolate. Since the data appears to be linear choose a method for determining a regression line. You can use

- the Mayer line method (shown here) or
- the Median-median method, or use
- the regression line from your calculator or
- draw it by hand

**Additional Resources:**

Visions Volume 1, Section 2.4, pp. 123 – 124.

<http://www.purplemath.com/modules/scattreg2.htm>



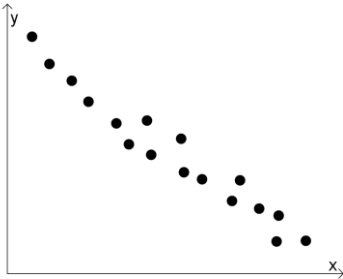


## **2.4 Interpretation of Linear Correlation**

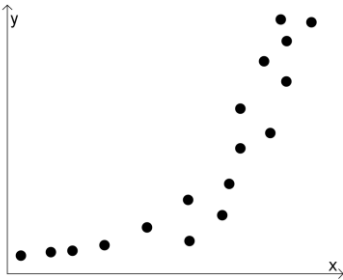
**Question:**

Which of the following distributions suggests a linear correlation of the data?

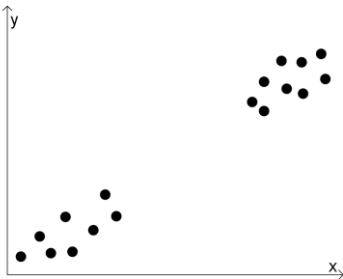
A)



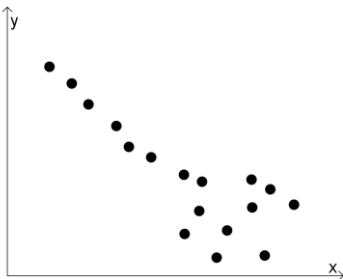
B)



C)



D)



**General Strategies:**

1. Read the question.
2. Highlight key words.
3. Identify the math topic.
4. Re-read the question.
5. Refer to your memory aid, as needed.
6. Look carefully at each choice shown (A, then B, then C and then D).
7. Eliminate options you know to be incorrect.
8. Solve/check each possible choice.
9. Select the choice that makes the most sense.

Do not leave a blank!

Make a choice!

**My Strategies:**

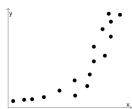
**Answer and Solution:**

A)



This data is clustered along a line

B)



This data is clustered along a curve (not linear)

C)



This data has a large gap and therefore a straight linear correlation can't be assumed

D)



This data is clustered in one area rather than along a line

**Suggested Strategies:**

The graph that shows data points that closely form a straight line yields the best interpretations for linear correlation

**The answer is A.**

**Additional Resources:**

Visions Volume 1, Section 2.4, pp. 123-124

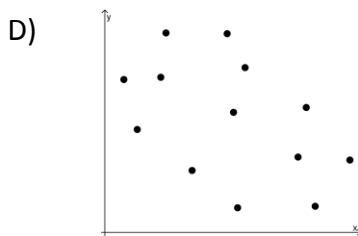
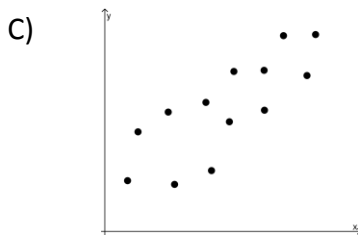
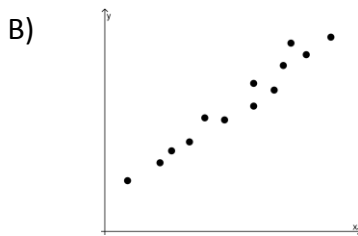
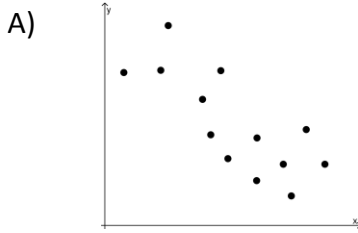
Khan Academy Video: <http://www.khanacademy.org/math/probability/regression/regression-correlation/v/fitting-a-line-to-data> see: Fitting a Line to Data, Estimating the Line of Best Fit

Explore Learning Gizmos, <http://www.explorelearning.com/> look up: Scatter Plots Activity A, Solving Using Trend Lines

**Question:**

Four different relationships between two variables are shown in the graphs below.

Which relationship would allow you to make the most accurate prediction?



**General Strategies:**

1. Read the question.
2. Highlight key words.
3. Identify the math topic.
4. Re-read the question.
5. Refer to your memory aid, as needed.
6. Look carefully at each choice shown (A, then B, then C and then D).
7. Eliminate options you know to be incorrect.
8. Solve/check each possible choice.
9. Select the choice that makes the most sense.

Do not leave a blank!  
Make a choice!

**My Strategies:**

**Answer and Solution:**

**B)** – the data points in this graph are closest to forming a line.

The answer is B.

**Suggested Strategies:**

The graph showing the strongest correlation, whether positive or negative, would demonstrate the strongest statistical link between two variables. A strong statistical link leads to better prediction.

**Additional Resources:**

Visions Volume 1, Section 2.4, pp. 123-124

Khan Academy Video: <http://www.khanacademy.org/math/probability/regression/regression-correlation/v/fitting-a-line-to-data> see: Fitting a Line to Data, Estimating the Line of Best Fit

Explore Learning Gizmos, <http://www.explorelearning.com/> look up: Scatter Plots Activity A, Solving Using Trend Lines

<http://www.purplemath.com/modules/scattreg2.htm>

**Question:**

The linear correlation coefficient between two variables is  $-0.93$ .

Which of the following best describes the correlation?

- A) The correlation between the two variables is strong and positive.
- B) The correlation between the two variables is strong and negative.
- C) The correlation between the two variables is weak and positive.
- D) The correlation between the two variables is weak and negative.

**General Strategies:**

1. Read the question.
2. Highlight key words.
3. Identify the math topic.
4. Re-read the question.
5. Refer to your memory aid, as needed.
6. Look carefully at each choice shown (A, then B, then C and then D).
7. Eliminate options you know to be incorrect.
8. Solve/check each possible choice.
9. Select the choice that makes the most sense.

Do not leave a blank!

Make a choice!

**My Strategies:**

**Answer and Solution:**

- A) False: The correlation between the two variables is strong and **not** positive.
- B) The correlation between the two variables is strong and negative.**
- C) False: The correlation between the two variables is **not** weak and positive.
- D) False: The correlation between the two variables is **not** weak and negative.

The answer is B.

**Suggested Strategies:**

The options are all wordy but you will see that they are almost exactly the same; look for the differences – you might want to use a highlighter...

The correlation coefficient of  $-0.93$  is close to  $-1$ , this results in a correlation described as negative and strong.

**Additional Resources:**

Visions Volume 1, Section 2.4, pp. 123-124

Khan Academy Video: <http://www.khanacademy.org/math/probability/regression/regression-correlation/v/fitting-a-line-to-data> see: Fitting a Line to Data, Estimating the Line of Best Fit

Explore Learning Gizmos, <http://www.explorelearning.com/> look up: Scatter Plots Activity A, Solving Using Trend Lines

<http://www.dummies.com/how-to/content/how-to-interpret-a-correlation-coefficient-r.html>

**Question:**

The linear correlation between two variables is positive and weak. Which of the following could represent the correlation coefficient?

- A) 0.32
- B) 0.87
- C) -0.26
- D) -0.91

**General Strategies:**

1. Read the question.
2. Highlight key words.
3. Identify the math topic.
4. Re-read the question.
5. Refer to your memory aid, as needed.
6. Look carefully at each choice shown (A, then B, then C and then D).
7. Eliminate options you know to be incorrect.
8. Solve/check each possible choice.
9. Select the choice that makes the most sense.

Do not leave a blank!  
Make a choice!

**My Strategies:**



**Answer and Solution:**

- A) **0.32 – close to 0 and positive**
- B) 0.87 – positive but not close to 0
- C) –0.26 – closest to 0 but negative
- D) –0.91 – negative and not close to 0

**Suggested Strategies:**

A weak and positive correlation is best represented by a correlation coefficient value that is positive and much closer to 0 than to 1.

**The answer is A.**

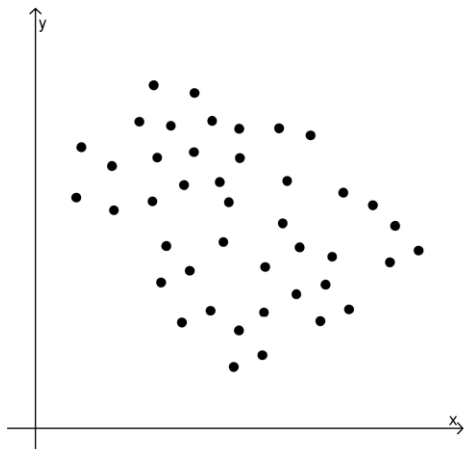
**Additional Resources:**

Visions Volume 1, Section 2.4, pp. 123-124

Khan Academy Video: <http://www.khanacademy.org/math/probability/regression/regression-correlation/v/fitting-a-line-to-data> see: Fitting a Line to Data, Estimating the Line of Best Fit  
Explore Learning Gizmos, <http://www.explorelearning.com/> look up: Scatter Plots Activity A, Solving Using Trend Lines

**Question:**

Consider the scatterplot below. Which of the following could best describe the linear correlation between the two variables?



- A) Strong and positive
- B) Strong and negative
- C) Weak and positive
- D) Weak and negative

**General Strategies:**

1. Read the question.
2. Highlight key words.
3. Identify the math topic.
4. Re-read the question.
5. Refer to your memory aid, as needed.
6. Look carefully at each choice shown (A, then B, then C and then D).
7. Eliminate options you know to be incorrect.
8. Solve/check each possible choice.
9. Select the choice that makes the most sense.

Do not leave a blank!  
Make a choice!

**My Strategies:**

**Answer and Solution:**

- A) Strong and positive
- B) Strong and negative
- C) Weak and positive
- D) Weak and negative**

**Specific Strategies:**

This scatterplot shows data points that trend downward suggesting a negative correlation.

This scatterplot also shows data points that are spread apart rather than close together (along a line) suggesting a weak correlation.

The answer is D.

**Additional Resources:**

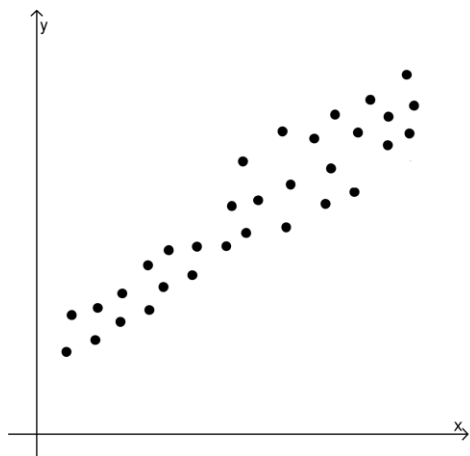
Visions Volume 1, Section 2.4, pp. 123-124

Khan Academy Video: <http://www.khanacademy.org/math/probability/regression/regression-correlation/v/fitting-a-line-to-data> see: Fitting a Line to Data, Estimating the Line of Best Fit

Explore Learning Gizmos, <http://www.explorellearning.com/> look up: Scatter Plots Activity A, Solving Using Trend Lines

**Question:**

Consider the scatterplot below. Which of the following could best describe the linear correlation coefficient between the two variables?



- A) 0.29
- B) 0.83
- C) -0.45
- D) -0.79

**General Strategies:**

1. Read the question.
2. Highlight key words.
3. Identify the math topic.
4. Re-read the question.
5. Refer to your memory aid, as needed.
6. Look carefully at each choice shown (A, then B, then C and then D).
7. Eliminate options you know to be incorrect.
8. Solve/check each possible choice.
9. Select the choice that makes the most sense.

Do not leave a blank!

Make a choice!

**My Strategies:**

**Answer and Solution:**

- A) 0.29 – positive but not high enough so suggest a strong correlation
- B) 0.83 – positive and high enough to suggest a strong correlation**
- C) –0.45 – a negative correlation, and not very strong
- D) –0.79 – a negative correlation, even though it is fairly strong.

**Suggested Strategies:**

This scatterplot shows data points that trend upward therefore you are looking for a positive correlation coefficient.

This scatterplot also shows data points that are close together (along a line) rather than spread out so you are looking for a correlation coefficient that suggests a strong correlation.

**The answer is B.**

**Additional Resources:**

Visions Volume 1, Section 2.4, pp. 123-124

Khan Academy Video: <http://www.khanacademy.org/math/probability/regression/regression-correlation/v/fitting-a-line-to-data> see: Fitting a Line to Data, Estimating the Line of Best Fit  
Explore Learning Gizmos, <http://www.explorelearning.com/> look up: Scatter Plots Activity A, Solving Using Trend Lines

**Question:**

A class of secondary 4 students measured their foot lengths and their heights. The results are represented in the table below.

**Data Collected**

Foot length (cm)	Height (cm)	Foot length (cm)	Height (cm)
22	154	25.5	170
22	151	25.5	173
23	155	26	167
23.5	165	27	174
24	160	27.5	175
24	158	28	176
24.5	165	28	183
25	161	28.5	185
25	163	29	190
25.5	164	29.5	186

Marco was absent was absent the day data was collected. School records show that Marco is 181 cm tall.

What is the predicted length of Marco's foot?

Marco's predicted foot length is \_\_\_\_\_.

**General Strategies:**

1. Read the problem.
2. Highlight key words.
3. Identify the math topics.
4. Re-read the problem.
5. Define your steps (your game plan) – this is criteria 3.
6. Refer to your memory aid, as needed.
7. Solve.
8. If you get stuck on a calculation, pick a number and keep going.
9. Ask yourself whether your answer makes sense.
10. Write your answer statement.

Show any or all your work! Do not leave a blank page!

**My Strategies:**

**Answer and Solution:**

With the Mayer line method:

$x$  represents the foot length in centimetres

$y$  represents the height in centimetres

Since the foot lengths are already in ascending order this step has been done for us.

$$x_1 = \frac{22+22+23+23.5+24+24+24.5+25+25+25.5}{10} = 23.85$$

$$x_2 = \frac{25.5+25.5+26+27+27.5+28+28+28.5+29+29.5}{10} = 27.45$$

$$y_1 = \frac{154+151+155+165+160+158+165+161+163+164}{10} = \frac{1596}{10} = 159.6$$

$$y_2 = \frac{170+173+167+174+175+176+183+185+190+186}{10} = \frac{1779}{10} = 177.9$$

$$a = \frac{177.9 - 159.6}{27.45 - 23.85} = \frac{183}{3.6} = 5.08$$

$$y = 5.08x + b \quad \text{using either point } (23.85, 159.6)$$

$$159.6 = 5.08(23.85) + b \quad \text{or } (27.45, 177.9)$$

$$159.6 = 121.16 + b$$

$$b = 38.44$$

$$y = 5.08x + 38.44$$

$$\text{Marco: } 181 = 5.08x + 38.44$$

$$181 - 38.44 = 5.08x$$

$$142.56 = 5.08x$$

$$x = \frac{142.56}{5.08} = 28 \text{ cm}$$

**Answer: Marco's predicted foot length is 28 cm.** (note that if you use a different method you will get a slightly different answer)

**Suggested Strategies:**

What you are looking for is a linear equation relating height and foot length. Once you've found one, you will use it to find foot length, knowing height.

There are a number of methods possible.

- Complete a scatter plot, drawing in the line of best fit and finding the equation of that line
- Use the median-median method
- Use the Mayer line method
- Enter the data into a graphing calculator to get the regression line

**Additional Resources:**

Visions Volume 1, Section 2.4, pp. 123-124

Explore Learning Gizmos, <http://www.explorelearning.com/> look up: Scatter Plots Activity A, Solving using trend lines

Khan Academy video: <http://www.khanacademy.org/math/probability/regression/regression-correlation/v/fitting-a-line-to-data> see fitting a line to data, estimating the line of best fit

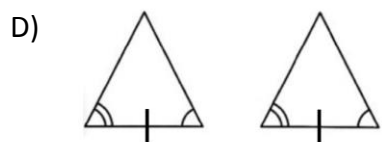
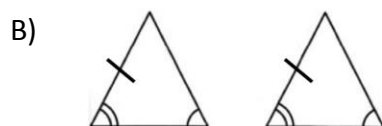
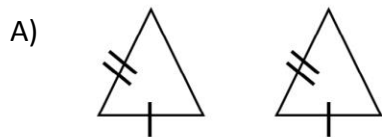




## **3.1 Congruent Triangles**

**Question:**

Which of the following pairs of triangles is not necessarily congruent?



**General Strategies:**

1. Read the question.
2. Highlight key words.
3. Identify the math topic.
4. Re-read the question.
5. Refer to your memory aid, as needed.
6. Look carefully at each choice shown (A, then B, then C and then D).
7. Eliminate options you know to be incorrect.
8. Solve/check each possible choice.
9. Select the choice that makes the most sense.

Do not leave a blank!  
Make a choice!

**My Strategies:**

**Answer and Solution:**

- A) **This is the correct answer. These triangles are *not necessarily* congruent since they only have two congruent sides and no congruent angles indicated.**
- B) These are congruent by ASA. Also, since two angles are congruent, then the third pair is too.
- C) These are congruent by SSS.
- D) These are congruent by ASA.

**Suggested Strategies:**

Check all possible answers, and beside each one write the proof that confirms congruency. You should be left with only one that has no proof (meaning those triangles are *not* congruent).

**The answer is A.**

**Additional Resources:**

Visions Volume 1, Section 3.1, pp. 149-151, pp. 160-161

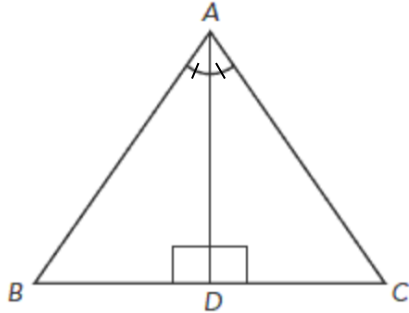
Explore Learning Gizmos, <http://www.explorelarning.com/> look up: Proving triangles congruent

Khan Academy video:

[http://www.khanacademy.org/search?page\\_search\\_query=congruent+triangles](http://www.khanacademy.org/search?page_search_query=congruent+triangles) Congruent triangles (all conditions: SSS, ASA and SAS)

**Question:**

Consider the following diagram.



What theorem can be used to show that  $\triangle ABD$  is necessarily congruent to  $\triangle ACD$ ?

- A) SSS
- B) SAS
- C) ASA
- D) AA

**General Strategies:**

1. Read the question.
2. Highlight key words.
3. Identify the math topic.
4. Re-read the question.
5. Refer to your memory aid, as needed.
6. Solve the problem without looking at choices shown (A, B, C and D).
7. Look at all the choices.
8. Match your answer to the appropriate choice.

Do not leave a blank!  
Make a choice!

**My Strategies:**

**Answer and Solution:**

The diagram identifies pairs of congruent angles and even though the congruent sides are not identified, the triangles share a side, which makes it congruent. Since the shared side is between pairs of congruent angles, the proof ASA is valid to prove congruency.

The answer is C.

**Suggested Strategies:**

When proving congruency ( $\cong$ ), first consider the three possible proofs (SSS, SAS, ASA). Starting with this will likely help you to eliminate one or two of the proofs as not having enough information pretty quickly.

Although this proof is perhaps not obvious since the congruent sides aren't identified, don't overlook the fact that the triangles share a side (AD).

**Additional Resources:**

Visions Volume 1, Section 3.1, pp. 149-151, pp. 160-161

Explore Learning Gizmos, <http://www.explorelearning.com/> look up: Proving triangles congruent

Khan Academy video:

[http://www.khanacademy.org/search?page\\_search\\_query=congruent+triangles](http://www.khanacademy.org/search?page_search_query=congruent+triangles) Congruent triangles (all conditions: SSS, ASA and SAS)

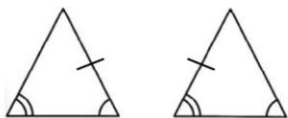
**Question**

Which of the following pairs of triangles is necessarily congruent?

A)



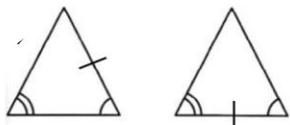
B)



C)



D)



**General Strategies:**

1. Read the question.
2. Highlight key words.
3. Identify the math topic.
4. Re-read the question.
5. Refer to your memory aid, as needed.
6. Look carefully at each choice shown (A, then B, then C and then D).
7. Eliminate options you know to be incorrect.
8. Solve/check each possible choice.
9. Select the choice that makes the most sense.

Do not leave a blank!

Make a choice!

**My Strategies:**

**Answer and Solution:**

Although the top angle is not identified as being congruent in the two triangles in answer b, they must be since the other two pairs of corresponding angles are the same. Since the unidentified angles are now known to be congruent, two theories can be used to prove congruency – ASA and SAS.

- A) There is not enough information, having two angles the same makes the triangle similar but not necessarily congruent
- B) This may look like ASA but the congruent sides are not between the congruent angles so you can't conclude the triangles are congruent by that theory.
- C) This is the correct answer.**
- D) This pair doesn't have corresponding sides that are congruent so you can't conclude they are congruent by ASA.

**The answer is C.**

**Suggested Strategies:**

Don't be discouraged if the correct answer doesn't jump out at you right away! Even though this is a multiple choice question, and you might expect to see the answer right away, there is often work or extra thinking needed to uncover the correct answer. Don't give up until you've tried all possibilities, in this case it involved a little extra thought.

**Additional Resources:**

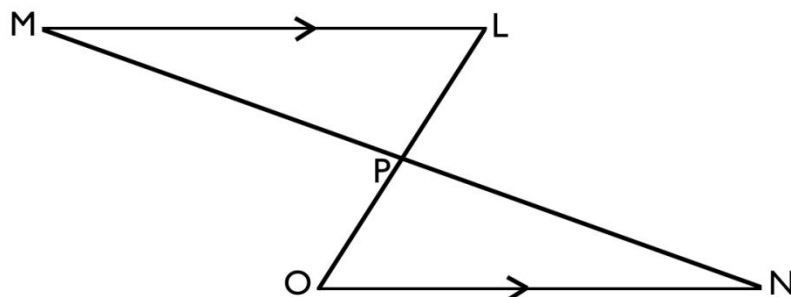
Visions Volume 1, Section 3.1, pp. 149-151, pp. 160-161

Explore Learning Gizmos, <http://www.explorellearning.com/> look up: Proving triangles congruent  
Khan Academy video:

[http://www.khanacademy.org/search?page\\_search\\_query=congruent+triangles](http://www.khanacademy.org/search?page_search_query=congruent+triangles) Congruent triangles (all conditions: SSS, ASA and SAS)

**Question:**

Point P is the midpoint of line segment MN. Segment ML is parallel to segment ON. Triangles  $\triangle LMP$  and  $\triangle ONP$  are congruent.



Show that triangles  $\triangle LMP$  and  $\triangle ONP$  are congruent. Remember to justify each of your statements.

\_\_\_\_\_  $\parallel$  \_\_\_\_\_

\_\_\_\_\_  $\parallel$  \_\_\_\_\_

\_\_\_\_\_  $\parallel$  \_\_\_\_\_

$\triangle LMP \cong \triangle ONP$  by \_\_\_\_\_

**General Strategies:**

1. Read the question.
2. Highlight key words.
3. Identify the math topic.
4. Re-read the question.
5. Make a prediction about the answer- what will it look like? (an equation, a number, etc.).
6. Refer to your memory aid, as needed.
7. Solve.
8. Ask yourself whether your answer makes sense.
9. Write your answer.

Do not leave a blank!

**My Strategies:**



**Answer and Solution:**

$\overline{MP} \cong \overline{NP}$  since P is the midpoint of MN, then MP and NP are congruent.

$\angle LPM \cong \angle OPN$   $\angle LPM$  and  $\angle OPN$  are vertically opposite and therefore congruent.

$\angle PML \cong \angle PNO$   $\angle PML$  and  $\angle PNO$  are alternate interior angles of a transversal through parallel lines, which means those angles are congruent.

With this information, we can say the triangles are necessarily congruent using the ASA theorem.

$$\overline{MP} \cong \overline{PN}$$

$$\angle LPM \cong \angle OPN$$

$$\angle PML \cong \angle PNO$$

$$\triangle LMP \cong \triangle ONP \text{ by } \underline{ASA}$$

**Suggested Strategies:**

Make sure you fill in all the information that you know on your diagrams. This is important on all questions of a test, but especially on ones where there is obviously information that has been left out. Don't forget all of the angle relationships when filling in information – and seeing a transversal through parallel lines should remind you of those angle relationships.

**Additional Resources:**

Visions Volume 1, Section 3.1, pp. 149-151, pp. 160-161

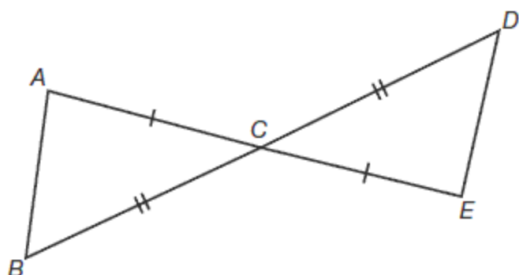
Explore Learning Gizmos, <http://www.explorellearning.com/> look up: Proving triangles congruent

Khan Academy video:

[http://www.khanacademy.org/search?page\\_search\\_query=congruent+triangles](http://www.khanacademy.org/search?page_search_query=congruent+triangles) Congruent triangles (all conditions: SSS, ASA and SAS)

**Question:**

Consider the following diagram.



Show that  $\triangle ABC$  is necessarily congruent to  $\triangle EDC$ .

\_\_\_\_\_  $\cong$  \_\_\_\_\_

\_\_\_\_\_  $\cong$  \_\_\_\_\_

\_\_\_\_\_  $\cong$  \_\_\_\_\_

$\triangle ABC \cong \triangle EDC$  by \_\_\_\_\_

**General Strategies:**

1. Read the question.
  2. Highlight key words.
  3. Identify the math topic.
  4. Re-read the question.
  5. Make a prediction about the answer- what will it look like? (an equation, a number, etc.).
  6. Refer to your memory aid, as needed.
  7. Solve.
  8. Ask yourself whether your answer makes sense.
  9. Write your answer.
- Do not leave a blank!

**My Strategies:**

**Answer and Solution:**

This diagram shows only two pairs of sides are congruent so you can eliminate ASA.

That leaves SSS and SAS. But you know that vertically opposite angles are necessarily congruent even if they aren't identified.

$$\overline{AC} \cong \overline{EC}$$

$$\angle ACB \cong \angle ECD$$

$$\overline{BC} \cong \overline{DC}$$

$$\triangle ABC \cong \triangle EDC \text{ by SAS}$$

**Suggested Strategies:**

When proving congruency consider the three possible proofs (SSS, SAS, ASA). Starting with this will likely help you to eliminate one or two of the proofs as not having enough information pretty quickly.

In other cases, don't forget to go through the possible angle relationships for intersecting and transverse lines across parallel lines.

**Additional Resources:**

Visions Volume 1, Section 3.1, pp. 149-151, pp. 160-161

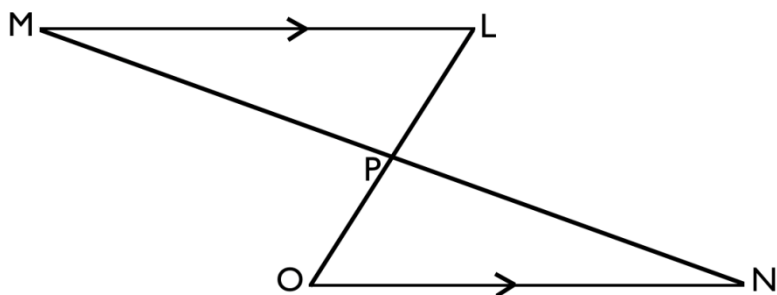
Explore Learning Gizmos, <http://www.explorelearning.com/> look up: Proving triangles congruent

Khan Academy video: [http://www.khanacademy.org/search?page\\_search\\_query=congruent+triangles](http://www.khanacademy.org/search?page_search_query=congruent+triangles)

Congruent triangles (all conditions: SSS, ASA and SAS)

**Question:**

You've been hired to paint a team logo on a field for a sporting event. The logo consists of two congruent triangles.



Here is the information you have:

- Point P is the midpoint of  $\overline{MN}$
- M is located at  $(-60, 48)$
- N is located at  $(28, 4)$
- $\angle LPM$  measures  $95^\circ$
- $\angle LMP$  measures  $15^\circ$

**\*\*measurements are in metres\*\***

You charge \$5 per  $m^2$  for doing the painting (round your measurements to the nearest  $m^2$ ).

What will you charge in total for painting the two triangles that make up the logo?

You will charge \_\_\_\_\_ for painting the two triangles that make up the logo.

**General Strategies:**

1. Read the problem.
2. Highlight key words.
3. Identify the math topics.
4. Re-read the problem.
5. Define your steps (your game plan) – this is criteria 3.
6. Refer to your memory aid, as needed.
7. Solve.
8. If you get stuck on a calculation, pick a number and keep going.
9. Ask yourself whether your answer makes sense.
10. Write your answer statement.

Show any or all your work! Do not leave a blank page!

**My Strategies:**

**Answer and Solution:**

Step 1) Calculate the coordinates of P using the midpoint formula. They are (-16, 26).

$$(x_m, y_m): \left( \frac{(-60+28)}{2}, \frac{(48+4)}{2} \right)$$

$$(x_m, y_m): \left( \frac{(-32)}{2}, \frac{(52)}{2} \right)$$

$$(x_m, y_m): (-16, 26)$$

**OR** Calculate the distance between the two points and divide that distance by 2.

Step 2) Calculate the distance from M to P. It is 49.1935 m.

$$d = \sqrt{(-16 - -60)^2 + (26 - 48)^2}$$

$$d = \sqrt{(44)^2 + (-22)^2}$$

$$d = \sqrt{1936 + 484}$$

$$d = \sqrt{2420}$$

$$d = 49.1935 \text{ m}$$

Step 3) The missing angle ( $\angle MLP$ ) is  $70^\circ$ . Using the Sine Law, calculate the missing measurements for  $\triangle LMP$ . We know side MP is 49.1935 m, we can calculate side LM. It is 52.15 m. We can then calculate side LP. It is 13.55 m.

$$\frac{49.1935}{\sin 70} = \frac{x}{\sin 15} = \frac{y}{\sin 95}$$

$$x = \frac{49.1935(\sin 15)}{\sin 70}$$

$$x = 13.55 \text{ metres (side LP)}$$

$$y = \frac{49.1935(\sin 95)}{\sin 70}$$

$$y = 52.15 \text{ m (side LM)}$$

Step 4) Calculate the area of  $\triangle LMP$  using either the Trig Formula or Hero's Formula. It is  $332 \text{ m}^2$  (rounded to the nearest square metre).

Example of Trig Formula:

$$\text{Area} = \frac{(13.55)(52.15)(\sin 70)}{2} = 332 \text{ m}^2$$

**OR** Hero's Formula:

$$\text{Half the perimeter: } \frac{49.19+13.55+52.15}{2} = 57.45$$

$$\text{Area} = \sqrt{57.45(57.45 - 49.19)(57.45 - 13.55)(57.45 - 52.15)}$$

$$\text{Area} = \sqrt{57.45(8.26)(43.9)(5.3)}$$

$$\text{Area} = \sqrt{110410.52379}$$

$$\text{Area} = 332 \text{ metres}^2$$

Step 5) Multiply the area of  $\triangle LMP$  by 2, then multiply that by \$5.

$$\text{The total cost of painting is } \$3320.$$

$$(332\text{m}^2 \times 2 \times \$5/\text{m}^2 = \$3320)$$

You will charge     \$3320     for painting the two triangles that make up the logo.

**Suggested Strategies:**

You need to find the area of the triangles and multiply that by \$5.

What do you need to find the area of a triangle?

- The length of a base and altitude or
- The length of two sides and the angle between them (Trig area formula) or
- The length of all three sides (Hero's Formula)

Choose the method you think will work for you and find the measures you need.

You only need to do this once since the triangles are congruent.

**Additional Resources:**

Visions Volume 1, Section 3.1, pp. 149-151, pp. 160-161

Khan Academy video: [http://www.khanacademy.org/search?page\\_search\\_query=congruent+triangles](http://www.khanacademy.org/search?page_search_query=congruent+triangles) Congruent triangles (all conditions: SSS, ASA and SAS)

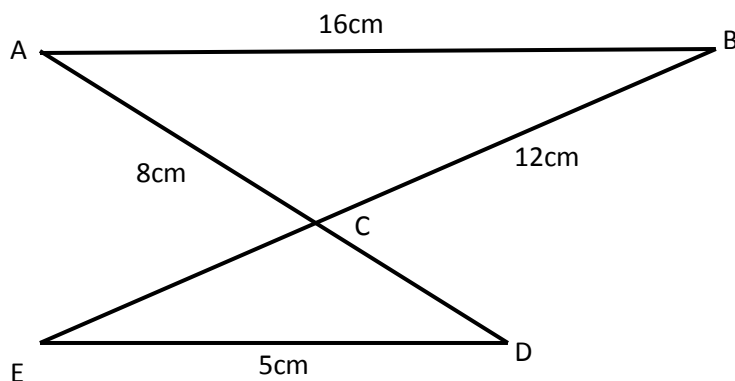
Explore Learning Gizmos, <http://www.explorellearning.com/> look up: Proving triangles congruent  
<http://mathbits.com/MathBits/TISection/Trig/AreaTrigTri.htm>



## **3.2 Similar Triangles**

**Question:**

The diagram below illustrates two triangles in which the measurements of some sides are given and  $\overline{AB} \parallel \overline{DE}$ .



What is the length of segment BE?

- A) 2.5 cm
- B) 3.75 cm
- C) 14.5 cm
- D) 15.75 cm

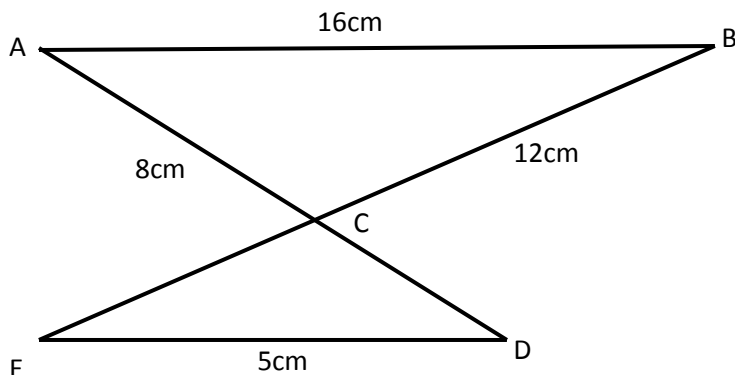
**General Strategies:**

1. Read the question.
2. Highlight key words.
3. Identify the math topic.
4. Re-read the question.
5. Refer to your memory aid, as needed.
6. Solve the problem without looking at choices shown (A, B, C and D).
7. Look at all the choices.
8. Match your answer to the appropriate choice.

Do not leave a blank!  
Make a choice!

**My Strategies:**



**Answer and Solution:**

$\triangle ABC \sim \triangle DEC$  because of the AA theorem – angle C is the same in both triangles, and because they are vertically opposite,  $\angle E \cong \angle B$  (and  $\angle D \cong \angle A$ ) because they are alternate interior angles.

In order to determine the total length of  $\overline{BE}$ , we need the length of  $\overline{CE}$ , so label the measure of  $\overline{CE}$  as “x”.

Since the triangles are similar, their sides must be proportional. Set up a proportion using corresponding sides:

$$\frac{m\overline{CE}}{m\overline{BC}} = \frac{m\overline{ED}}{m\overline{AB}} \rightarrow \frac{x}{12} = \frac{5}{16}$$

Cross multiply to determine the value of x

$$x = 12 \times 5 \div 16 = 3.75 \text{ cm}$$

To determine the length of side BE, add  $m\overline{BC} + m\overline{CE}$

$$12 + 3.75 = 15.75 \text{ cm}$$

- A) 2.5 cm is the measure of segment CD.
- B) 3.75 cm is the measure of segment EC. The measure of segment BC must be added to this.
- C) The result of adding the measures of segments CD and BC instead of EC and BC.
- D) **15.75 cm is correct.**

The answer is D.

**Specific Strategies:**

Some distractors might stand out: choices A) and B) are both fairly small, whereas C) and D) are both fairly large. An educated guess would eliminate A) and B), but we should remember that the drawings are never to scale.

- Label the figure with the given measurements
- Recognize that the triangles are similar because of AA

*Alternate strategy:*

*Determine the scale factor k by dividing the lengths of corresponding sides:  $k = 3.2$*

*Divide side BC by 3.2 to get the length of side  $CE = 3.75 \text{ cm}$*

- After completing the calculations, re-read the question and re-read the choices

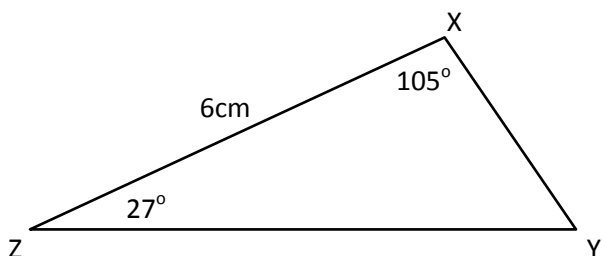
**Additional Resources:**

Visions Volume 1, Section 3.2, p. 171 (Minimum Conditions for Similar Triangles)

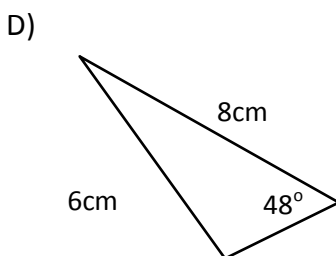
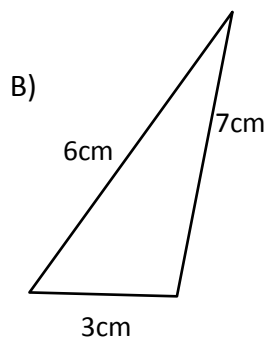
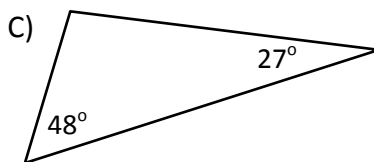
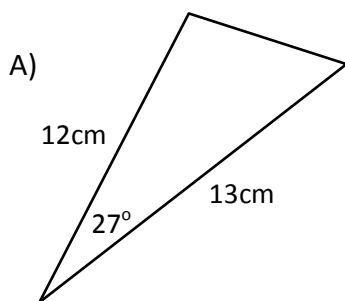
Khan Academy video: <http://www.khanacademy.org/math/geometry/similarity/triangle-similarity/v/similarity-example-problems> Similarity example problems

**Question:**

Triangle XYZ is shown below.



Which of the triangles below is necessarily similar to triangle XYZ?



**General Strategies:**

1. Read the question
2. Highlight key words
3. Identify the math topic
4. Re-read the question
5. Refer to your memory aid, as needed
6. Look carefully at each choice shown (A, then B, then C and then D)
7. Eliminate options you know to be incorrect
8. Solve/check each possible choice
9. Select the choice that makes the most sense

Do not leave a blank! Make a choice!

**My Strategies:**

**Answer and Solution:**

$m\angle y = 180^\circ - 105^\circ - 27^\circ = 48^\circ$  since the angles in every triangle add up to  $180^\circ$

- A) Incorrect: This triangle only has one angle in common with triangle XYZ. Two side measures are given, but it is only possible to compare with one side of XYZ and we need at least two sides to prove SSS or SAS. (If you used the Sine Law to find the missing side of the original triangle, you will also see that the sides are not proportional.)
- B) Incorrect: This triangle gives us three side measurements, but like option A, we would need to be able to compare at least two sides to prove SSS or SAS.
- C) Correct: This triangle is similar to XYZ. It has two angles in common because the third angle was calculated above to be  $48^\circ$ . So by AA, the triangles are similar.**
- D) Incorrect: This triangle is not necessarily similar to XYZ because the information the triangle is not unique – you can make many triangles with those three features fixed.

The answer is C.

**Specific Strategies:**

- Keyword: Similar
- Recall theorems on similar triangles
- Determine the measure of the third angle in triangle XYZ
- Triangles are similar if they satisfy one of three theorems – AA, SSS or SAS

**Additional Resources:**

Visions Volume 1, Section 3.2, p. 171 (Minimum Conditions for Similar Triangles)  
 Khan Academy video: [http://www.khanacademy.org/math/geometry/similarity/triangle\\_similarity/v/similarity-example-problems](http://www.khanacademy.org/math/geometry/similarity/triangle_similarity/v/similarity-example-problems) Similarity example problems

**Question:**

In the diagram below,  $\overline{BD}$  and  $\overline{AE}$  intersect at C

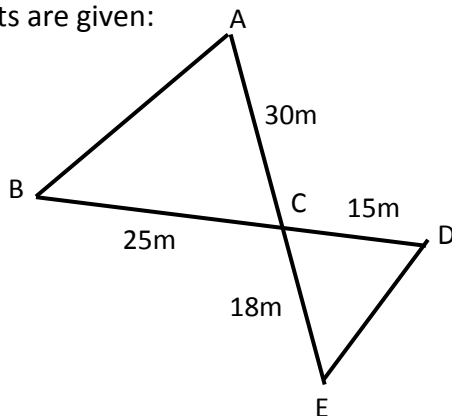
Other measurements are given:

$$m\overline{AC} = 30m$$

$$m\overline{BC} = 25m$$

$$m\overline{CD} = 15m$$

$$m\overline{CE} = 18m$$



Which of the following statements could be used to prove that triangle ABC is similar to triangle EDC?

- A) Two triangles are similar if they have two congruent corresponding angles. (AA)
- B) Two triangles are similar if they have corresponding sides that are proportional. (SSS)
- C) Two triangles are similar if they have corresponding proportional contained between two congruent angles. (ASA)
- D) Two triangles are similar if they have a congruent angle contained between the corresponding sides that are proportional. (SAS)

**General Strategies:**

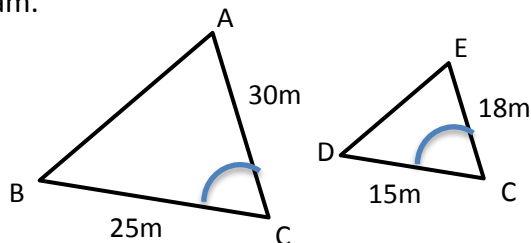
1. Read the question
2. Highlight key words
3. Identify the math topic
4. Re-read the question
5. Refer to your memory aid, as needed
6. Look carefully at each choice shown (A, then B, then C and then D)
7. Eliminate options you know to be incorrect
8. Solve/check each possible choice
9. Select the choice that makes the most sense

Do not leave a blank! Make a choice!

**My Strategies:**

**Answer and Solution:**

$\angle C \cong \angle C$  because they are vertically opposite angles in the original diagram.



Because two corresponding pairs of sides are given, as well as the angle contained, we should consider that the triangles are similar by SAS.

Check if the sides are proportional:

$$\checkmark \frac{30}{18} = \frac{25}{15}$$

- A) We are only given one pair of corresponding angles and we don't know for sure that side  $\overline{AB}$  is parallel to side  $\overline{DE}$ , so it's not AA.
- B) Only 3 side lengths are given so we cannot consider SSS.
- C) ASA is not a theorem for similarity.
- D) **Because the proportions of two pairs of sides are the same and the angle contained by those sides is congruent, the triangles are similar by SAS.**

The answer is D.

**Specific Strategies:**

- Keyword: Similar
- Recall theorems on similar triangles
- Re-draw the triangles in the same orientation – *be careful with the rotation*
- Take note of other measurements which are not given in the problem
- Consider each of the similarity theorems

**Additional Resources:**

Visions Volume 1, Section 3.2, p. 171 (Minimum Conditions for Similar Triangles)

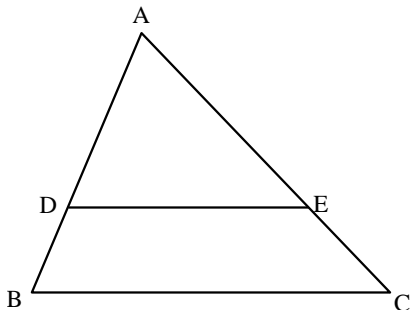
Khan Academy video: <http://www.khanacademy.org/math/geometry/similarity/triangle-similarity/v/similarity-example-problems> Similarity example problems

Khan Academy video: <http://www.khanacademy.org/math/geometry/similarity/triangle-similarity/v/similar-triangle-example-problems> Similar triangle example problems

**Question:**

In the figure below, triangles ABC and ADE are similar.

$\overline{DE}$  is parallel to  $\overline{BC}$ ,  
 $m\overline{DE} = 20$  m,  
 $m\overline{BC} = 35$  m  
and  
 $m\overline{AE} = 24$  m.



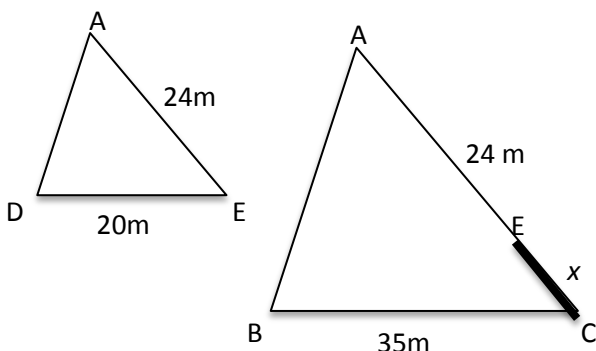
What is the length of segment EC?

**General Strategies:**

1. Read the question.
  2. Highlight key words.
  3. Identify the math topic.
  4. Re-read the question.
  5. Make a prediction about the answer- what will it look like? (an equation, a number, etc.).
  6. Refer to your memory aid, as needed.
  7. Solve.
  8. Ask yourself whether your answer makes sense.
  9. Write your answer.
- Do not leave a blank!

**My Strategies:**

The length of segment EC is \_\_\_\_\_ m.

**Answer and Solution:**

Because the unknown length is part of side  $\overline{AC}$ , it will be simpler to solve for the length of side  $\overline{AC}$  first and then subtract 24m.

Set up a proportion using corresponding sides:

$$\frac{m\overline{DE}}{m\overline{BC}} = \frac{m\overline{AE}}{m\overline{AC}}$$

$$\frac{20}{35} = \frac{24}{m\overline{AC}}$$

$$m\overline{AC} = 42$$

If side AC measures 42 m, then the length of EC is:

$$42 - 24 = 18 \text{ m}$$

**Answer: The length of segment EC is 18 m.**

**Specific Strategies:**

- Keyword: Similar
- Label measurements directly on the figure

Highlight the length that the question is asking for

- Re-draw the triangles separately with the same orientation and include any measurements given
- Highlight corresponding sides with different colors
- Label the unknown measurement as "x"
- Reminder: re-do your calculations to double check, you will not get partial credit if your final answer is wrong!

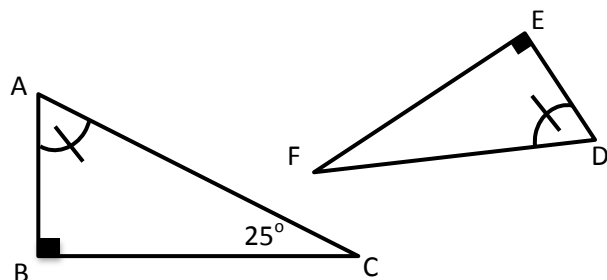
**Additional Resources:**

Visions Volume 1, section 3.2, p. 171 (Minimum Conditions for Similar Triangles)

Khan Academy video: [http://www.khanacademy.org/math/geometry/similarity/triangle\\_similarity/v/similarity-example-problems](http://www.khanacademy.org/math/geometry/similarity/triangle_similarity/v/similarity-example-problems) Similarity example problems

**Question:**

The following information is given about right triangles ABC and DEF:



What is the measure of  $\angle D$ ?

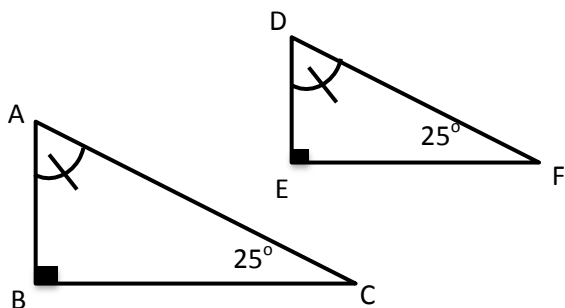
The measure of  $\angle D$  is \_\_\_\_\_.

**General Strategies:**

1. Read the question.
  2. Highlight key words.
  3. Identify the math topic.
  4. Re-read the question.
  5. Make a prediction about the answer- what will it look like? (an equation, a number, etc.).
  6. Refer to your memory aid, as needed.
  7. Solve.
  8. Ask yourself whether your answer makes sense.
  9. Write your answer.
- Do not leave a blank!

**My Strategies:**



**Answer and Solution:**

$$m\angle D = 180^\circ - 90^\circ - 25^\circ = 65^\circ$$

**Answer:** The measure of  $\angle D$  is  $65^\circ$ .

**Specific Strategies:**

Label the given measurements on the figure

- Label the given measurements on the figure
- Draw the figures side by side in the same orientation
- Because the triangles have two angles in common, they must be similar by the AA theorem
- If two triangles have two angles in common, their third angle must also be common.

**Additional Resources:**

Visions Volume 1, Section 3.2, p. 171 – Minimum Conditions for Similar Triangles

Khan Academy video: [http://www.khanacademy.org/math/geometry/similarity/triangle\\_similarity/v/similar-triangle-basics](http://www.khanacademy.org/math/geometry/similarity/triangle_similarity/v/similar-triangle-basics) Similar triangles basics

**Question:**

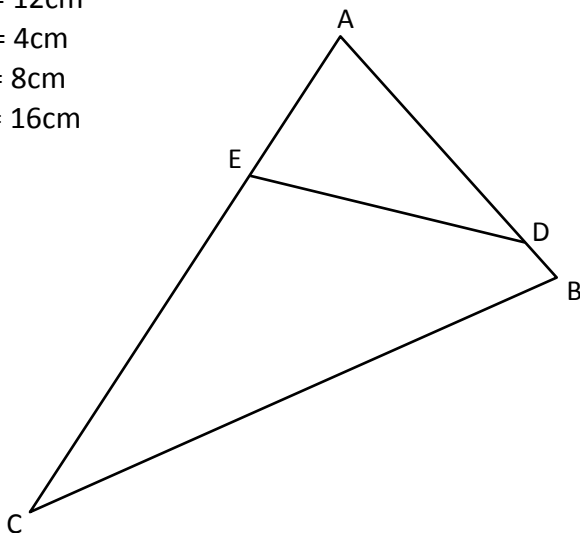
The following measures are given for the figure below:

$$m\overline{AD} = 12\text{cm}$$

$$m\overline{DB} = 4\text{cm}$$

$$m\overline{AE} = 8\text{cm}$$

$$m\overline{EC} = 16\text{cm}$$



Is triangle  $\triangle ABC$  similar to  $\triangle AED$ ?

Note: The figure is not necessarily drawn to scale.

Yes, triangle  $\triangle ABC$  is similar to  $\triangle AED$

No, triangle  $\triangle ABC$  is not similar to  $\triangle AED$

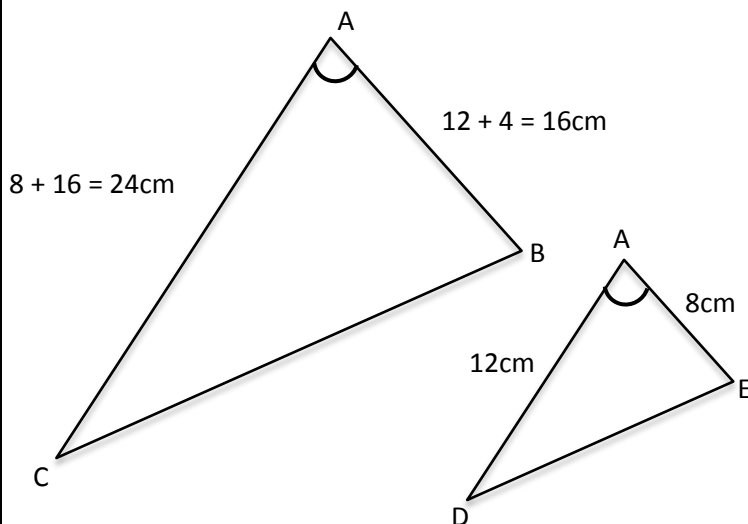
**General Strategies:**

1. Read the problem.
2. Highlight key words.
3. Identify the math topics.
4. Re-read the problem.
5. Define your steps (your game plan) – this is criteria 3.
6. Refer to your memory aid, as needed.
7. Solve.
8. If you get stuck on a calculation, pick a number and keep going.
9. Ask yourself whether your answer makes sense.
10. Write your answer statement.

Show any or all your work! Do not leave a blank page!

**My Strategies:**

**Answer and Solution:**



Statement	Justification
The ratio of one pair of corresponding sides = 2	$\frac{m\overline{AC}}{m\overline{AD}} = \frac{24\text{cm}}{12\text{cm}} = 2$
$\angle A \cong \angle A$	Corresponding angles are congruent
The ratio of a second pair of corresponding sides = 2	$\frac{m\overline{AB}}{m\overline{AE}} = \frac{16\text{cm}}{8\text{cm}} = 2$
$\triangle ABC \sim \triangle AED$	SAS (Two triangles that have one congruent angle contained between corresponding sides of proportional length are similar)

- Yes, triangle  $\triangle ABC$  is similar to  $\triangle AED$
- No, triangle  $\triangle ABC$  is not similar to  $\triangle AED$

**Specific Strategies:**

- Label the figure with the given measurements
- Highlight the task
- Re-draw the triangles separately, with the same orientation
- Label the figure with the given measurements
- Label any other known measurements – we know that angle  $A$  measures the same in both triangles
- Highlight corresponding sides of the triangles
- Determine which geometric statement will allow you to prove the triangles are similar – SAS

**Additional Resources:**

Visions Volume 1, Section 3.2, p. 171 (Minimum Conditions for Similar Triangles)  
 Khan Academy video: <http://www.khanacademy.org/math/geometry/similarity/triangle-similarity/v/similarity-postulates> Similarity postulates  
 Explore Learning Gizmos, <http://www.explorelearning.com/> look up Similarity in Right Triangles

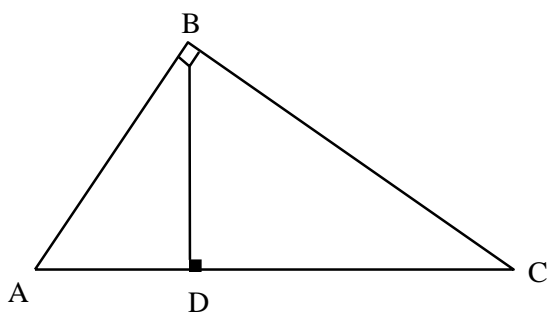


## **3.3 Metric Relations (Right Triangles)**

**Question:**

The following information about triangle ABC below is known:

- $m\angle ABC = 90^\circ$
- $\overline{BD}$  is an altitude
- $m\overline{AB} = 60$  m
- $m\overline{BC} = 80$  m



What is the measure of altitude  $\overline{BD}$ ?

- A) 36 m
- B) 48 m
- C) 64 m
- D) 69 m

**General Strategies:**

1. Read the question.
2. Highlight key words.
3. Identify the math topic.
4. Re-read the question.
5. Refer to your memory aid, as needed.
6. Solve the problem without looking at choices shown (A, B, C and D).
7. Look at all the choices.
8. Match your answer to the appropriate choice.

Do not leave a blank!  
Make a choice!

**My Strategies:**

**Answer and Solution:**

Step 1: Pythagorean Theorem

$$(\overline{mAC})^2 = 60^2 + 80^2$$

$$\overline{mAC} = \sqrt{3600 + 6400}$$

$$\overline{mAC} = 100 \text{ m}$$

Step 2: Apply Metric Relation

$$a \cdot b = c \cdot h$$

$$(\overline{mCB})(\overline{mAB}) = (\overline{mAC})(\overline{mBD})$$

$$60(80) = 100(\overline{mBD})$$

$$\overline{mBD} = \frac{4800}{100}$$

$$\overline{mBD} = 48 \text{ m}$$

- A) 36 m - this is  $\overline{mAD}$
- B) 48 m - correct**
- C) 64 m - this is  $\overline{mDC}$
- D) 69 m - this is the result from using an incorrect formula

**The answer is B.**

**Suggested Strategies:**

- 1) Put the measures onto the diagram.
- 2) If you are using formulas, make sure you label the triangle according to your formulas.
- 3) Identify the metric relation(s) that enable(s) you to solve for the unknown.
- 4) Consider that it might be more than one step.

*Note: In this case you must apply Pythagorean theorem first before applying a metric relation formula.*

**Additional Resources:**

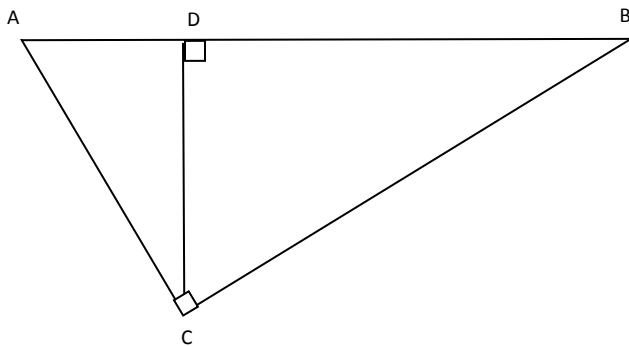
Visions Volume 1, Section 3.3, p. 181

Explore Learning Gizmos, <http://www.explorellearning.com/> look up: similarity in right triangles

**Question:**

Triangle ABC has the following properties:

- $m\angle ACB = 90^\circ$
- $\overline{DC}$  is an altitude
- $m\overline{DB} = 45$  m
- $m\overline{AD} = 12$  m



What is the area of  $\triangle ABC$ ?

- A)  $139 \text{ m}^2$
- B)  $523 \text{ m}^2$
- C)  $662 \text{ m}^2$
- D)  $1325 \text{ m}^2$

**General Strategies:**

1. Read the question.
2. Highlight key words.
3. Identify the math topic.
4. Re-read the question.
5. Refer to your memory aid, as needed.
6. Solve the problem without looking at choices shown (A, B, C and D).
7. Look at all the choices.
8. Match your answer to the appropriate choice.

Do not leave a blank!  
Make a choice!

**My Strategies:**



**Answer and Solution:**

Step 1: Metric Relation

$$\begin{aligned}
 h^2 &= m \cdot n \\
 (m\overline{CD})^2 &= (m\overline{AD}) \cdot (m\overline{DB}) \\
 m\overline{CD} &= \sqrt{12(45)} \\
 m\overline{CD} &= \sqrt{540} \\
 m\overline{CD} &= 23.2379 \text{ meters}
 \end{aligned}$$

Step 2: Area of Triangle ABC

$$\begin{aligned}
 &\frac{m\overline{CD}(m\overline{AD} + m\overline{DB})}{2} \\
 &\frac{23.2379(12 + 45)}{2} \\
 &662m^2
 \end{aligned}$$

The answer is C.

**Suggested Strategies:**

- 1) Orient the triangle in a way that is easiest for you.
- 2) Put the measures on the diagram and re-label if necessary.
- 3) Select the appropriate metric relation formula. *In this case we need 'h' and we are given 'm' and 'n'.*
- 4) Remember your basic area formulas. In this case we need area of triangle.

$$Area = \frac{base \times height}{2}$$

*Also be sure you get the area of the requested triangle. In this case the questions ask for triangle ABC – the largest of the three triangles.*

**Additional Resources:**

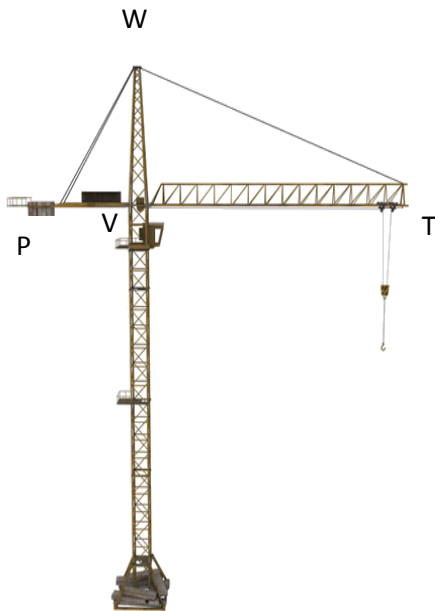
Visions Volume 1, Section 3.3, p. 181

Explore Learning Gizmos, <http://www.explorelarning.com/> look up: similarity in right triangles  
<http://www.youtube.com/watch?v=fdEBhf9SOYA>

**Question:**

A construction crane pictured below, has the following measurements:

- $m\angle TWP = 90^\circ$
- $\overline{VW}$  is an altitude
- $m\overline{VW} = 50$  metres
- $m\overline{TV} = 70$  metres



What is the measure of angle WPV to the nearest tenth of a degree?

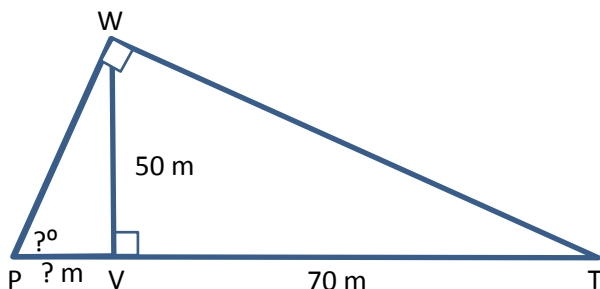
The measure of angle WPV is \_\_\_\_\_.

**General Strategies:**

1. Read the question.
  2. Highlight key words.
  3. Identify the math topic.
  4. Re-read the question.
  5. Make a prediction about the answer- what will it look like? (an equation, a number, etc.).
  6. Refer to your memory aid, as needed.
  7. Solve.
  8. Ask yourself whether your answer makes sense.
  9. Write your answer.
- Do not leave a blank!

**My Strategies:**

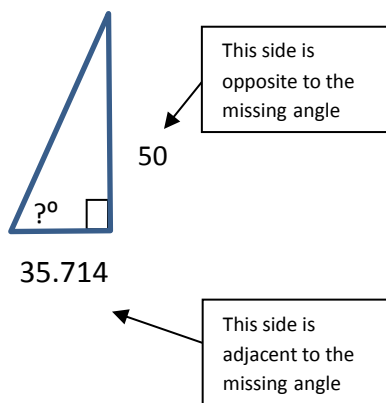
**Answer and Solution:**



Use Metric Relations to find the missing side.

$$\begin{aligned}
 h^2 &= m \cdot n \\
 50^2 &= 70(m\overline{PV}) \\
 \frac{50^2}{70} &= m\overline{PV} \\
 \frac{250}{7} &= m\overline{PV} \\
 m\overline{PV} &= 35.714
 \end{aligned}$$

Use a Trigonometric Ratio to find the missing angle.



$$\tan^{-1}\left(\frac{50}{35.714}\right) = 54.46^\circ$$

The measure of angle WPV is ≈ 54.5°.

**Suggested Strategies:**

- 1) Redraw the diagram with only the triangles and measures (so the crane doesn't confuse you.) Now locate angle WPV.
- 2) If we find the measure of PV, we can use a trig ratio (tangent) to find the missing angle.
- 3) To find the missing measure we'll use a metric relation that uses the measures we have.

Make sure your calculator is set to degree mode.

**Additional Resources:**

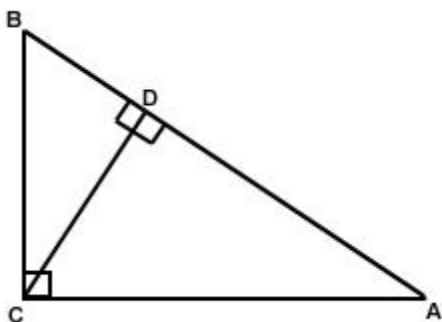
Visions Volume 1, Section 3.3, p. 181

Explore Learning Gizmos, <http://www.explorelearning.com/> look up: similarity in right triangles

**Question:**

Triangle ABC has the following properties:

- $m\angle BCA = 90^\circ$
- $\overline{CD}$  is an altitude
- $m\overline{AB} = 20$  m
- $m\overline{BC} = 10$  m



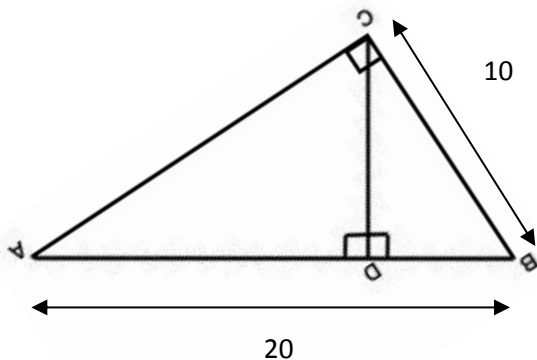
What is the measure of  $\overline{AD}$ ?

The measure of  $\overline{DA}$  is \_\_\_\_\_.

**General Strategies:**

1. Read the question.
  2. Highlight key words.
  3. Identify the math topic.
  4. Re-read the question.
  5. Make a prediction about the answer- what will it look like? (an equation, a number, etc.).
  6. Refer to your memory aid, as needed.
  7. Solve.
  8. Ask yourself whether your answer makes sense.
  9. Write your answer.
- Do not leave a blank!

**My Strategies:**

**Answer and Solution:**

Step 1: Solve for  $m\overline{DB}$

$$\begin{aligned}
 a^2 &= n \cdot c \\
 (m\overline{BC})^2 &= (m\overline{BA})(m\overline{DB}) \\
 10^2 &= 20(m\overline{DB}) \\
 \frac{100}{20} &= (m\overline{DB}) \\
 (m\overline{DB}) &= 5
 \end{aligned}$$

Step 2: Solve for  $(m\overline{AD})$

$$\begin{aligned}
 (m\overline{AB}) &= m\overline{AD} + m\overline{DB} \\
 20 &= m\overline{AD} + 5 \\
 \mathbf{m\overline{AD}} &= \mathbf{15}
 \end{aligned}$$

The measure of  $\overline{AD}$  is 15 m.

**Suggested Strategies:**

- 1) If necessary, orient the triangle in a way that is easiest for you.
- 2) If necessary, re-label the vertices of triangle according to your memory aid.
- 3) Add the numerical information to the diagram.
- 4) Identify the metric relation formula that enables you to solve for the unknown or set up a proportion knowing the three triangles are similar:

$$\frac{hyp_{small}}{short\ side_{small}} = \frac{hyp_{large}}{short\ side_{large}}$$

$$\frac{10}{\overline{BD}} = \frac{20}{10}$$

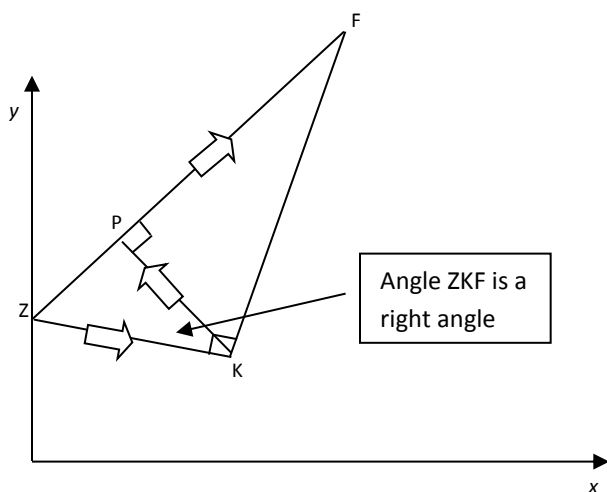
**Additional Resources:**

Visions Volume 1, Section 3.3, p. 181

Explore Learning Gizmos, <http://www.explorelearning.com/> look up: similarity in right triangles

**Question:**

A group of Brazilian soccer players are practicing their passes before a game. Their coach illustrates on a Cartesian plane a possible game scenario by showing the players as vertices of three similar right angle triangles.



The coach places the player Zico (Z) on the sideline (y-axis) to perform a throw-in to the player Kaka (K), who would pass the ball to Pele (P) located at the coordinates (15, 60) followed by a pass to Falcao (F) located at (60, 90). Units are in metres.

What is the total combined distance of all three passes?

The total combined distance is \_\_\_\_\_ m.

**General Strategies:**

1. Read the problem.
2. Highlight key words.
3. Identify the math topics.
4. Re-read the problem.
5. Define your steps (your game plan) – this is criteria 3.
6. Refer to your memory aid, as needed.
7. Solve.
8. If you get stuck on a calculation, pick a number and keep going.
9. Ask yourself whether your answer makes sense.
10. Write your answer statement.

Show any or all your work! Do not leave a blank page!

**My Strategies:**

**Answer and Solution:**

Step 1: Solve  $m\overline{PF}$

Distance formula  $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$   
 $P(15, 60)$  and  $F(60, 90)$   $\sqrt{(60 - 15)^2 + (90 - 60)^2} = m\overline{PF}$   
 $m\overline{PF} = \sqrt{45^2 + 30^2} = 54.08$

Step 2: Solve  $m\overline{ZP}$

Set-up equation of line ZF  $y = ax + b$   
 Given  $P(15, 60)$  and  $F(60, 90)$

$$a = \frac{y_2 - y_1}{x_2 - x_1} = \frac{90 - 60}{60 - 15} = \frac{30}{45} = \frac{2}{3}$$

$$y = \frac{2}{3}x + b \quad \text{Now we have to solve for 'b'.$$

Solve for 'b' by substituting the coordinates of a point on the line into the equation. In this case we have a choice between point P and F. It does not matter which one you choose.

Using the coordinates of point P we have,

$$60 = \frac{2}{3}(15) + b$$

$$60 = 10 + b$$

$$b = 50$$

This is the y-coordinate of point Z

Step 3: Solve  $m\overline{PZ}$

Distance formula  $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$   
 $Z(0, 50)$  and  $P(15, 60)$   
 $\sqrt{(15 - 0)^2 + (60 - 50)^2} = m\overline{PZ}$   
 $m\overline{PZ} = \sqrt{15^2 + 10^2} = 18.03$

Step 4: Metric Relations to solve  $m\overline{PK}$

$$h^2 = m \cdot n$$

$$(m\overline{PK})^2 = (m\overline{ZP})(m\overline{PF})$$

$$m\overline{PK} = \sqrt{18.03(54.08)}$$

$$m\overline{PK} = 31.23$$

Step 5: Solving for  $m\overline{ZK}$

$$m\overline{ZK} = \sqrt{(m\overline{ZP})^2 + (m\overline{PK})^2}$$

$$m\overline{ZK} = \sqrt{18.03^2 + 31.23^2}$$

$$m\overline{ZK} = 36.06$$

Step 6: Sum up the lengths of three line segments

$$m\overline{ZK} + m\overline{PK} + m\overline{PF} = 121.37 \text{ meters}$$

**The total combined distance is 121.37 m.**

**Suggested Strategies:**

- 1) We know we have to find distances of three line segments. Line segment  $\overline{PF}$  can be found with the distance formula since we are given the coordinates of point P and point F.
- 2) We know Point Z is on the y-axis. This means the x-coordinate is zero. To solve for the y-coordinate we can set up an equation of a line.

$$y = ax + b$$

Recall: 'a' in the equation represents the slope  $\frac{y_2 - y_1}{x_2 - x_1}$  and 'b' represent the y-intercept.

**Additional Resources:**

Visions Volume 1, Section 3.3, p. 181

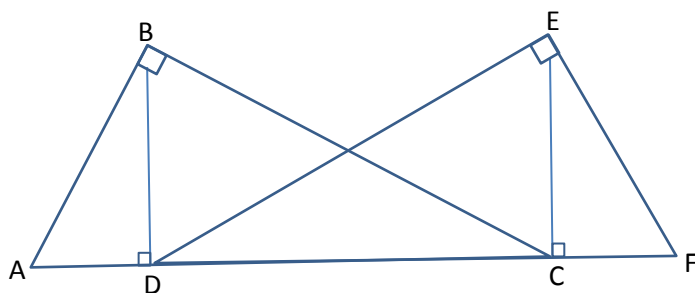
Explore Learning Gizmos, <http://www.explorelearning.com/> look up: similarity in right triangles

**Question:**

A group of engineers is planning the construction of the new Champlain Bridge in Montreal. Below is a diagram of a section of the bridge.

The bridge's towers ( $\overline{BD}$  and  $\overline{EC}$ ) are each 100 metres in height and one of the support cables ( $\overline{AB}$ ) measures 110 metres.

Also,  $m\angle ABC$  and  $m\angle DEF$  are both  $90^\circ$  and the towers are perpendicular to the base of the bridge.



Show that the length of the cable represented by segment DE measures 240 m.

**General Strategies:**

1. Read the problem.
2. Highlight key words.
3. Identify the math topics.
4. Re-read the problem.
5. Define your steps (your game plan) – this is criteria 3.
6. Refer to your memory aid, as needed.
7. Solve.
8. If you get stuck on a calculation, pick a number and keep going.
9. Ask yourself whether your answer makes sense.
10. Write your answer statement.

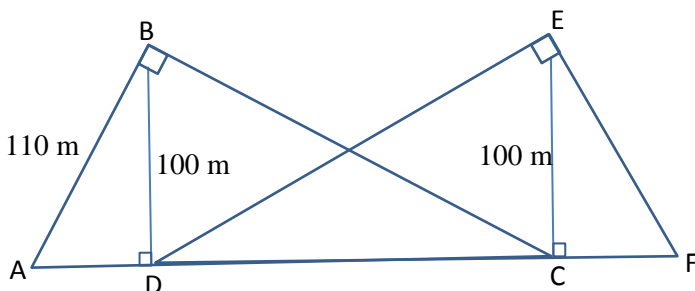
Show any or all your work! Do not leave a blank page!

**My Strategies:**



**Answer and Solution:**

Here is an example of an appropriate method



Step 1: We can prove that  $\triangle BCD$  and  $\triangle ECD$  are congruent by theorem SAS:

Justification:

- $m\overline{CD}$  is a shared length to both triangles
- $m\overline{BD} = m\overline{CE}$  Given information
- $m\angle ABC$  and  $m\angle DEF$  are both  $90^\circ$

$m\overline{BC} = m\overline{DE}$  since they are corresponding sides of congruent triangles.

Step 2: Apply Pythagorean Theorem to find  $m\overline{AD}$

$$m\overline{AD} = \sqrt{110^2 - 100^2}$$

$$m\overline{AD} = 45.8258$$

Step 3: Apply Metric Relation to find  $m\overline{DC}$

$$h^2 = m \cdot n$$

$$(m\overline{BD})^2 = (m\overline{AD})(m\overline{CD})$$

$$100^2 = 45.8258(m\overline{CD})$$

$$\frac{100^2}{45.8258} = (m\overline{CD})$$

$$(m\overline{CD}) = 218.2177$$

Step 4: Apply Pythagorean Theorem to find  $m\overline{BC}$

$$(m\overline{BC}) = \sqrt{218.2177^2 + 100^2}$$

$$(m\overline{BC}) = 240 \text{ metres}$$

The length of the cable represented by  $\overline{DE}$  is indeed 240 m.

**Suggested Strategies**

- 1) Put the numerical information into the diagram.
- 2) Since triangle DEF has no information about it and triangle DEC only has one measure, convince yourself that the triangles with more measures can be used.
- 3) See that  $\overline{BC}$  is the same length as  $\overline{DE}$  so if you find  $m\overline{BC}$  you'll know  $m\overline{DE}$ .
- 4) Use Pythagoras to find  $m\overline{AD}$ .
- 5) Use a metric relation to find  $m\overline{DC}$ .
- 6) Use Pythagoras again to find  $m\overline{BC}$ .
- 7)  $\overline{BC}$  is the same length as  $\overline{DE}$ .

**Additional Resources:**

Visions Volume 1, Section 3.3, p. 181

Explore Learning Gizmos, <http://www.explorelearning.com/> look up: similarity in right triangles



## **4.1 Real Functions**

**Question:**

In a laboratory, scientists are recording the growth rate of cells. They report that a sample of 50 cells doubled every hour.

Which of the following rules describes the relationship?

- A)  $f(x) = 50(x^2)$
- B)  $f(x) = 50(2^x)$
- C)  $f(x) = 50 + 2x$
- D)  $f(x) = 2(50^x)$

**General Strategies:**

1. Read the question.
2. Highlight key words.
3. Identify the math topic.
4. Re-read the question.
5. Refer to your memory aid, as needed.
6. Look carefully at each choice shown (A, then B, then C and then D).
7. Eliminate options you know to be incorrect.
8. Solve/check each possible choice.
9. Select the choice that makes the most sense.

Do not leave a blank!

Make a choice!

**My Strategies:**

**Answer and Solution:**

$x$ (number of hours)	$f(x)$ (total number of cells)
0	50
1	$50 \times 2 = 100$
2	$100 \times 2 = 200$
3	$200 \times 2 = 400$

An initial amount which increases by the same multiplier is an exponential function

$$f(x) = a(c^x)$$

- A)  $f(x) = 50(x^2)$   
Since we see the term  $x^2$ , this is a quadratic or second-degree polynomial function.
- B)  **$f(x) = 50(2^x)$**   
**This is an exponential function with an initial value of 50 and a base of 2, which means that it is multiplying by 2 for each increase in  $x$ .**
- C)  $f(x) = 50 + 2x$   
This is a linear function or first-degree polynomial function since we see no exponents.
- D)  $f(x) = 2(50^x)$   
This is an exponential function, but the initial value and the base are switched.

**The answer is B.**

**Specific Strategies:**

- Since the choices are different function rules; the goal of the problem is to translate the situation into a functional model.
- Make a table of values to get a clearer picture of the relation
- Doubling the number of cells means you must multiply by 2 for each hour
- Look on your memory aid for the rule of an exponential function

**Additional Resources:**

Visions Volume 2, Section 4.1, p. 17 (Families of Functions)

Visions Volume 2, Section 4.3, p. 39 (Exponential Functions)

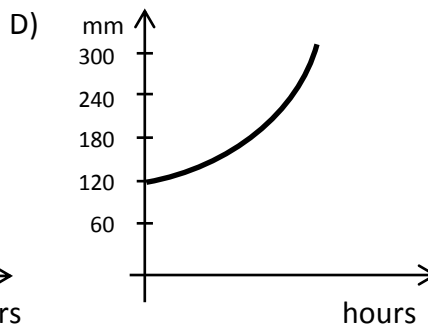
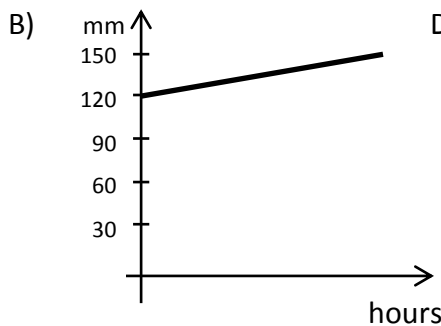
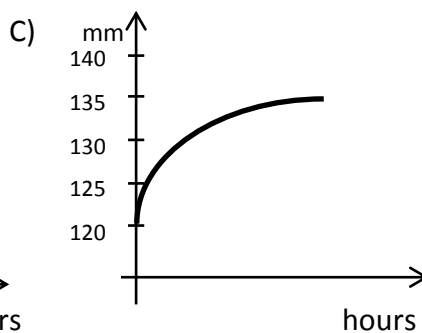
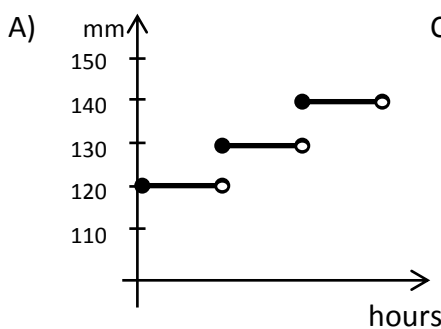
Explore Learning Gizmos, <http://www.explorelearning.com/> look up: Exponential functions – Activity A

**Question:**

A small town in Quebec already received 120 mm of rain this year when a severe storm occurred. During the storm, rain fell at a constant rate of 5 mm per hour.

The graphs below relate the number of hours since the storm began with the accumulated rainfall in mm.

Which graph below correctly illustrates the relationship?



**General Strategies:**

1. Read the question.
2. Highlight key words.
3. Identify the math topic.
4. Re-read the question.
5. Refer to your memory aid, as needed.
6. Look carefully at each choice shown (A, then B, then C and then D).
7. Eliminate options you know to be incorrect.
8. Solve/check each possible choice.
9. Select the choice that makes the most sense.

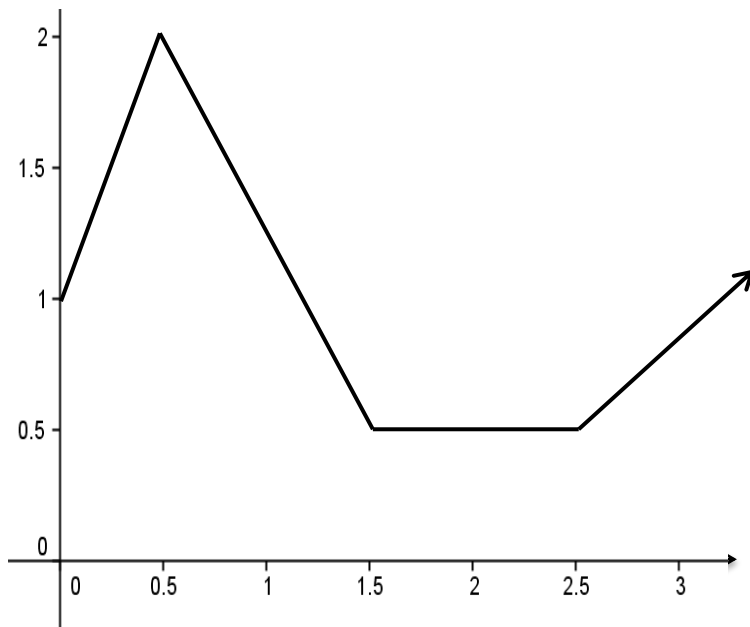
Do not leave a blank!  
Make a choice!

**My Strategies:**

<p><b>Answer and Solution:</b></p> <p>A) This option is a step function, which means that the amount of rain accumulated remains constant for a period of time and jumps abruptly at critical values; it does not make sense to represent a constant increase of 5mm per hour</p> <p><b>B) This option is a linear function of first-degree polynomial function. Its initial value is 120 and it has a positive slope, indicating a constant increase</b></p> <p>C) This option does not illustrate a constant increase; the amount of rain increases sharply at first, then accumulates more slowly</p> <p>D) This option appears to be an exponential function, which would mean that the amount of rain increases by some multiplying factor instead of a constant rate</p> <p><b>The answer is B.</b></p>	<p><b>Specific Strategies:</b></p> <p>Since the choices are graphs, this is a problem that can be answered by observation; no calculations need to be done.</p> <ul style="list-style-type: none"><li>- Consider all the functional models on your memory aid</li><li>- The linear function model is the only one which offers an interpretation for a constant increase over time</li></ul>
<p><b>Additional Resources:</b></p> <p>Visions Volume 1, Section 1.2, p. 22 (Lines in the Cartesian Plane) Visions Volume 2, Section 4.1, p. 17 (Families of Functions) Khan Academy video: <a href="http://www.khanacademy.org/math/algebra/algebra-functions/relationships_functions/v/basic-linear-function">http://www.khanacademy.org/math/algebra/algebra-functions/relationships_functions/v/basic-linear-function</a> Ex. Constructing a Function</p>	

**Question:**

The graph below illustrates a piecewise function whose domain is  $[0, +\infty[$ .



Which of the following statements is TRUE?

- A) The function has no  $x$ -intercept.
- B) The function has no  $y$ -intercept.
- C) The function is negative over the interval  $[0.5, 1.5]$ .
- D) The function has no extrema.

**General Strategies:**

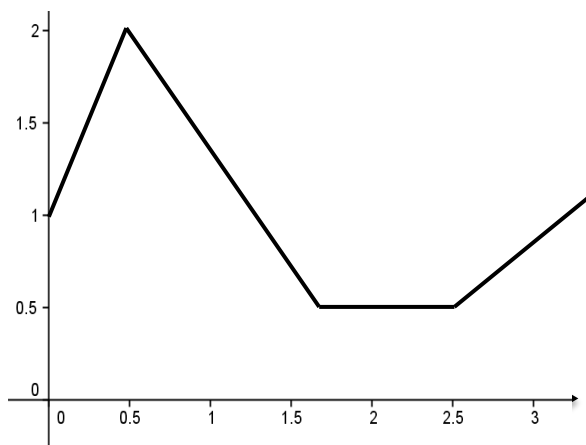
1. Read the question.
2. Highlight key words.
3. Identify the math topic.
4. Re-read the question.
5. Refer to your memory aid, as needed.
6. Look carefully at each choice shown (A, then B, then C and then D).
7. Eliminate options you know to be incorrect.
8. Solve/check each possible choice.
9. Select the choice that makes the most sense.

Do not leave a blank!

Make a choice!

**My Strategies:**



**Answer and Solution:**

- A) **The function continues to increase after  $x = 2.5$  and it never touches the  $x$ -axis before that so there is no  $x$ -intercept**
- B) The  $y$ -intercept of this function is where  $x = 0$  and therefore  $y = 1$
- C) The function is decreasing, meaning that it goes down from left to right over the interval  $[0.5, 1.5]$
- D) Since the function continues to increase after  $x = 2.5$ , it has no maximum, but it definitely has a minimum at  $y$ -value of 0.5

**The answer is A.**

**Specific Strategies:**

Since the answers are properties of functions, we need to analyze the graph to determine the answer. No calculations are necessary.

- Find the definitions for properties of functions on your memory aid
- Make sure you know which properties concern  $x$  values and which properties concern  $y$ -values

***x-values:*** domain,  $x$ -intercept intervals of increase and decrease, positive and negative intervals

***y-values:*** range,  $y$ -intercept, extrema (maximum and minimum)

**Additional Resources:**

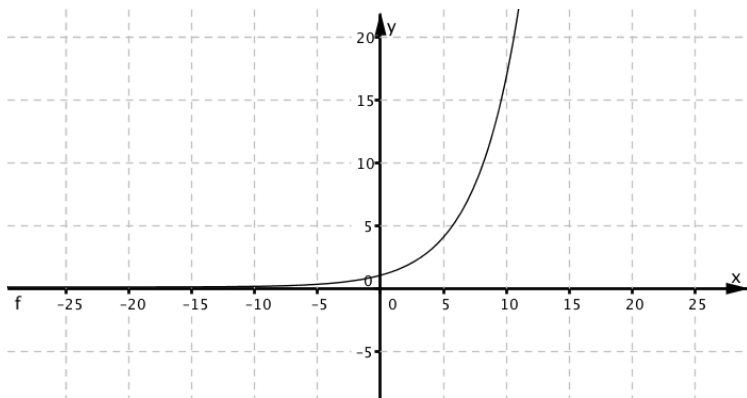
Visions Volume 2, Section 4, (Revision), pp. 7-8 (Properties of Functions)

Khan Academy video: [Comparing features of functions 2 \(example 1\)](#) ; [Interpreting features of functions 2 \(example 1\)](#) ; [When a function is positive or negative](#)

Khan Academy video: [http://www.khanacademy.org/math/trigonometry/functions\\_and\\_graphs/analyzing\\_functions/v/when-a-function-is-positive-or-negative](http://www.khanacademy.org/math/trigonometry/functions_and_graphs/analyzing_functions/v/when-a-function-is-positive-or-negative)

**Question:**

Which of the rules below corresponds to the following graph?



A)  $f(x) = 4x + 3$

B)  $f(x) = \left(\frac{4}{3}\right)^x$

C)  $f(x) = 4x^2 + 3$

D)  $f(x) = \frac{43}{x}$

**General Strategies:**

1. Read the question.
2. Highlight key words.
3. Identify the math topic.
4. Re-read the question.
5. Refer to your memory aid, as needed.
6. Look carefully at each choice shown (A, then B, then C and then D).
7. Eliminate options you know to be incorrect.
8. Solve/check each possible choice.
9. Select the choice that makes the most sense.

Do not leave a blank!

Make a choice!

**My Strategies:**

**Answer and Solution:**

- A) The  $x$  only has a coefficient so this is a linear function and would be a line.
- B) **Correct - The  $x$  is an exponent so this is an exponential function and would curve up (or down) sharply and cross the  $y$ -axis.**
- C) The  $x$  is squared so this is a quadratic function and would be a U-shape.
- D) The  $x$  is a denominator so this is an inverse (rational) function and, in this particular case, would not cross the  $y$ -axis (since  $x \neq 0$ ).

**The answer is B.**

**Suggested Strategies:**

- Compare the graph of the functions studied and their corresponding rules, to the ones provided in the question.

**Additional Resources:**

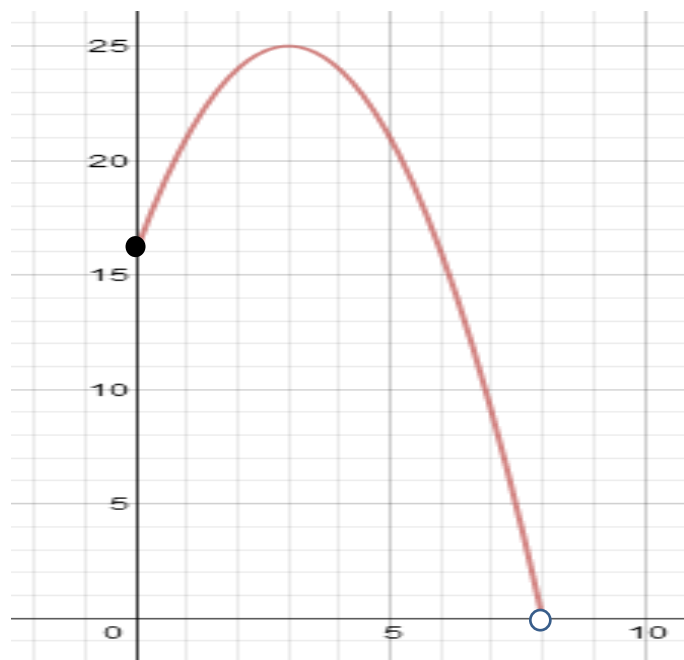
Visions Volume 2, Section 4.1, p. 17 (Families of Functions)

Visions Volume 2, Section 4.3, p. 39 (Exponential Functions)

Khan Academy video; [http://www.khanacademy.org/math/trigonometry/exponential\\_and\\_logarithmic\\_func/exp\\_growth\\_decay/v/exponential-growth-functions](http://www.khanacademy.org/math/trigonometry/exponential_and_logarithmic_func/exp_growth_decay/v/exponential-growth-functions) – Exponential grown functions

**Question:**

What is the range of the function below?



The range of the function is \_\_\_\_\_.

**General Strategies:**

1. Read the question.
  2. Highlight key words.
  3. Identify the math topic.
  4. Re-read the question.
  5. Make a prediction about the answer- what will it look like? (an equation, a number, etc.).
  6. Refer to your memory aid, as needed.
  7. Solve.
  8. Ask yourself whether your answer makes sense.
  9. Write your answer.
- Do not leave a blank!

**My Strategies:**

**Answer and Solution:**

The range is the set of all possible values of  $y$  from least to greatest (bottom to top).

The lowest value on the graph is  $y = 0$  but this point is an open circle (and therefore not included) and the greatest value on the graph is  $y = 25$ .

Therefore, the range is  $]0, 25]$ .

**Specific Strategies:**

Look for the lowest value of  $y$  on the graph and the highest value of  $y$  on the graph.

Use interval notation in providing your answer. (Remember that the smallest value comes first.)

**Answer:** The range of the function is  $]0, 25]$ .

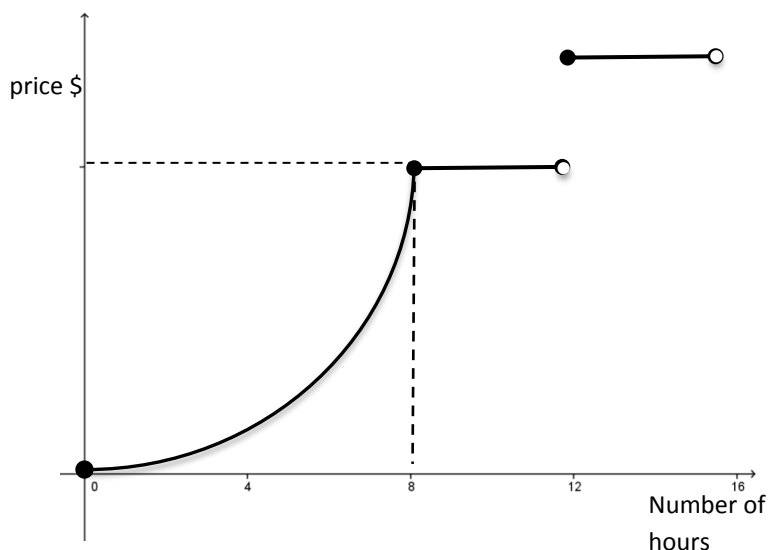
**Additional Resources:**

Visions Volume 2, Section 4 (Revision), p. 7 (Properties of Functions)

**Question:**

A yard and garden care contractor has developed a mathematical model to determine the price he will charge his clients throughout the season. In order to get his clients interested in his service, he gradually increases his price per hour as the hours accumulate.

He illustrates this model in the graph below.



The first piece of the function is a second-degree polynomial function given by the following rule:

$$g(x) = 10x^2 \quad \text{where } 0 \leq x \leq 8$$

The price will remain constant for the next 4 hours but after 12 hours, the contractor charges a flat rate of \$250 for every four hours of work or part thereof.

One client is charged \$1640.

What are the possible numbers of hours that job would have taken?

That job would have taken between \_\_\_\_ and \_\_\_\_ hours.

**General Strategies:**

1. Read the problem.
2. Highlight key words.
3. Identify the math topics.
4. Re-read the problem.
5. Define your steps (your game plan) – this is criteria 3.
6. Refer to your memory aid, as needed.
7. Solve.
8. If you get stuck on a calculation, pick a number and keep going.
9. Ask yourself whether your answer makes sense.
10. Write your answer statement.

Show any or all your work! Do not leave a blank page!

**My Strategies:**

**Answer and Solution:**

Determine the y-value when  $x = 8$  for the function  $g(x)$

$$g(8) = 10(8)^2$$

*BEDMAS: Remember to apply the exponent first*

$$\begin{aligned} g(8) &= 10 \times 64 \\ &= 640 \end{aligned}$$

So, the step function begins at a y-value of \$640 after 8 hours of work. The price doesn't go up until 12 hours.

x (Number of hours)	y (Cost \$)
[8, 12[	\$640
[12, 16[	\$640 + 250 = \$890
[16, 20[	\$890 + 250 = \$1140
[20, 24[	\$1140 + 250 = 1390
<b>[24, 28[</b>	<b>\$1390 + 250 = \$1640</b>

For \$1640, it will take 24 to 28 hours, not including 28 hours.

**That job would have taken between 24 and 28 hours.**

**Specific Strategies:**

- The first unknown that needs to be determined is the cost for 8 hours of work
- Make sure to keep your work organized from this point – a table is a great idea
- According to the step function, the open circle at  $x = 12$  is a critical value, so the cost will “jump” starting at 12 hours

**Additional Resources:**

Visions Volume 2, Section 4.1, p. 17 (Families of Functions)

Visions Volume 2, Section 4.2, p. 28 (Second-degree Polynomial Function)

Visions Volume 2, Section 4.3, p. 53 (Piecewise Function)

Step Graphs: <http://www.youtube.com/watch?v=LUshzsvoGZU>





## **4.2 Second-Degree Polynomial Function**

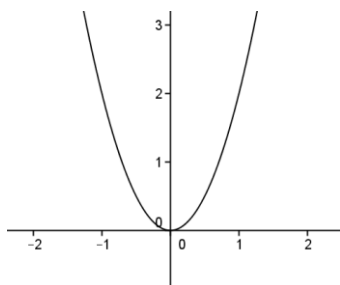
**Question:**

Consider the following function:

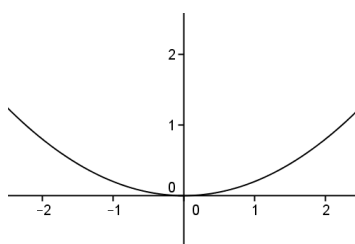
$$f(x) = 2x^2$$

Which of the following graphs represents the function?

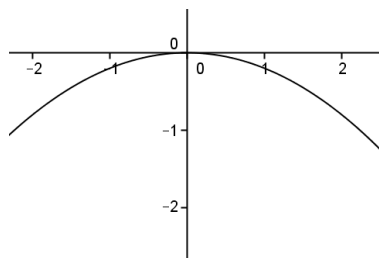
A)



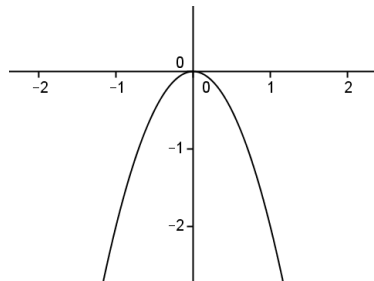
B)



C)



D)

**General Strategies:**

1. Read the question.
2. Highlight key words.
3. Identify the math topic.
4. Re-read the question.
5. Refer to your memory aid, as needed.
6. Look carefully at each choice shown (A, then B, then C and then D).
7. Eliminate options you know to be incorrect.
8. Solve/check each possible choice.
9. Select the choice that makes the most sense.

Do not leave a blank!

Make a choice!

**My Strategies:**

**Answer and Solution:**

- A) **This parabola opens upward, so it must have a positive “a” value. Also, when  $x = 1$  we can check the y-value by using the rule:**  
 $f(x) = 2(1)^2 = 2$ . **Since the parabola appears to pass through the point (1, 2), we can conclude that this is the correct graph.**
- B) This parabola opens upward, so it must have a positive “a”. However, when  $x = 1$ , the value of the function is clearly less than 1, which does not satisfy the equation  $f(x) = 2x^2$ .
- C) This parabola opens downward, so parameter “a” is a negative value.
- D) This parabola opens downward, so parameter “a” is a negative value.

**Specific Strategies:**

Remember:  
 For a quadratic or second-degree polynomial function,  $y = ax^2$

Since parameter “a” is positive, the parabola should open upward.

Since parameter “a” is longer than 1, the parabola should be narrower than the basic function  $f(x) = x^2$ .

**The answer is A.**

**Additional Resources:**

Visions Volume 2, pp. 28-29

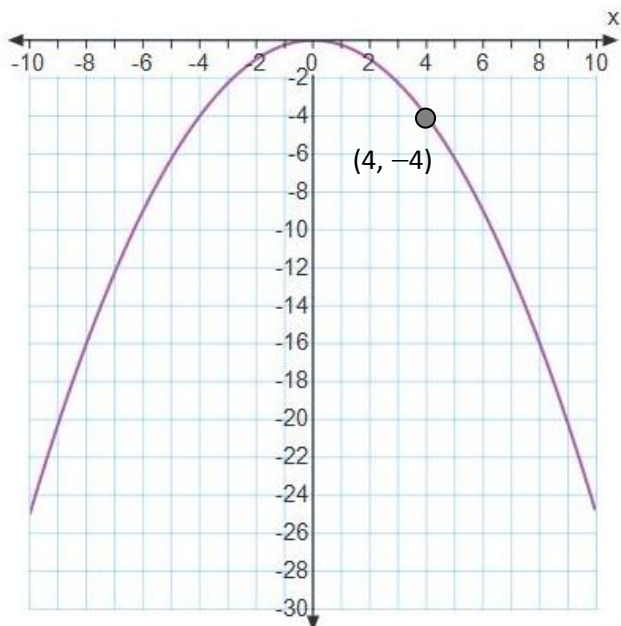
Khan Academy video: [https://www.khanacademy.org/math/algebra/quadratics/solving\\_graphing\\_quadratics/v/graphing-a-quadratic-function](https://www.khanacademy.org/math/algebra/quadratics/solving_graphing_quadratics/v/graphing-a-quadratic-function)

<http://www.purplemath.com/modules/grphquad.htm>

Explore Learning Gizmos, <http://www.explorelearning.com/> look up: Quadratics in Polynomial Form- Activity A and Activity B (for these, you must keep the b and c sliders at position 0)

**Question:**

Which rule represents the following graph?



- A)  $y = 0.25x^2$
- B)  $y = -0.25x^2$
- C)  $y = -0.25^x$
- D)  $y = 0.25x + 1$

**General Strategies:**

1. Read the question.
2. Highlight key words.
3. Identify the math topic.
4. Re-read the question.
5. Refer to your memory aid, as needed.
6. Look carefully at each choice shown (A, then B, then C and then D).
7. Eliminate options you know to be incorrect.
8. Solve/check each possible choice.
9. Select the choice that makes the most sense.

Do not leave a blank!

Make a choice!

**My Strategies:**

**Answer and Solution:**

Just by looking... only B matches the graph

- A) quadratic opening up  
**B) quadratic opening down**  
 C) exponential function  
 D) linear function

Confirm by testing the point...

- A)  $-4 = 0.25(4)^2$   
 $-4 = 4$  *false*
- B)  $-4 = -0.25(4)^2$**   
 $-4 = -4$  *true*
- C)  $-4 = -0.25^4$   
 $-4 = 0.0039$  *false*
- D)  $-4 = 0.25(4) + 1$   
 $-4 = 2$  *false*

Since only one statement is true, that must be the answer.

**The answer is B** (a quadratic with a negative parameter “a”).

**Suggested Strategies:**

Imagine the shape of each type of function.

- A quadratic ( $ax^2$ ) looks like a U.
- An exponential ( $ac^x$ ) is a curve that either increases or decreases over the whole domain.
- A linear ( $ax + b$ ) is a straight line.

Recall for a quadratic function if:

- Parameter “a” is positive the parabola opens upward → U is up- Happy Face!
- Parameter “a” is negative the parabola opens downward → U is down- Sad Face!

**Additional Resources:**

Visions Volume 2, pp. 28-29

Khan Academy video: [https://www.khanacademy.org/math/algebra/quadratics/solving\\_graphing\\_quadratics/v/graphing-a-quadratic-function](https://www.khanacademy.org/math/algebra/quadratics/solving_graphing_quadratics/v/graphing-a-quadratic-function)

<http://www.purplemath.com/modules/grphquad.htm>

Explore Learning Gizmos, <http://www.explorelearning.com/> look up: Quadratics in Polynomial Form- Activity A and Activity B (for these, you must keep the b and c sliders at position 0)

**Question:**

Consider the following table of values for a quadratic function.

$x$	$f(x)$
-5	7.5
0	0
5	7.5
10	30
15	67.5
20	120

Which of the following rules represents the quadratic function?

- A)  $f(x) = -3x^2$
- B)  $f(x) = -0.3x^2$
- C)  $f(x) = 0.3x^2$
- D)  $f(x) = 3x^2$

**General Strategies:**

1. Read the question.
2. Highlight key words.
3. Identify the math topic.
4. Re-read the question.
5. Refer to your memory aid, as needed.
6. Look carefully at each choice shown (A, then B, then C and then D).
7. Eliminate options you know to be incorrect.
8. Solve/check each possible choice.
9. Select the choice that makes the most sense.

Do not leave a blank!  
Make a choice!

**My Strategies:**

**Answer and Solution:**

- A)  $f(x) = -3x^2$   
This function opens down (decreases after the vertex) but the table of values increases after the vertex
- B)  $f(x) = -0.3x^2$   
This function opens down (decreases after the vertex) but the table of values increases after the vertex
- C)  $f(x) = 0.3x^2$   
**This function opens up and when a non-zero x-value is tested, it gives the corresponding value for f(x):**  
 $f(5) = 0.3(5)^2 = 0.3 \times 25 = 7.5$
- D)  $f(x) = 3x^2$   
This function opens up but when a non-zero x-value is tested, it doesn't give the corresponding f(x).  
 $f(5) = 3(5)^2 = 3 \times 25 = 75 \neq 7.5$

**OR**

Calculate parameter "a" algebraically using one of the points given:

$$f(x) = ax^2$$

for example: (5, 7.5)

$$\begin{aligned} 7.5 &= a(5)^2 \\ \frac{7.5}{25} &= \frac{a \times 25}{25} \\ 0.3 &= a \end{aligned}$$

**The answer is C.****Suggested Strategies:**

- See what patterns you notice among the data.
  - Symmetry around (0,0)
  - (0, 0) is a minimum so the "a" value will be positive.
  - Since you are told it is quadratic, you know it's in the form  $ax^2$  so you can calculate a.

Test the two positive functions to see which one works.

**Additional Resources:**

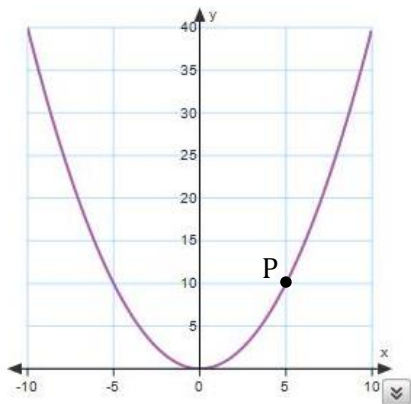
Visions Volume 2, pp. 28-29

Khan Academy video: [https://www.khanacademy.org/math/algebra/quadratics/solving\\_graphing\\_quadratics/v/graphing-a-quadratic-function](https://www.khanacademy.org/math/algebra/quadratics/solving_graphing_quadratics/v/graphing-a-quadratic-function)<http://www.purplemath.com/modules/grphquad.htm>Explore Learning Gizmos, <http://www.explorelearning.com/> look up: Quadratics in Polynomial Form- Activity A and Activity B (for these, you must keep the b and c sliders at position 0)

**Question:**

Point P (5, 10) is on the curve of the 2<sup>nd</sup> degree function below.

What is the rule of the function?

**General Strategies:**

1. Read the question.
  2. Highlight key words.
  3. Identify the math topic.
  4. Re-read the question.
  5. Make a prediction about the answer- what will it look like? (an equation, a number, etc.).
  6. Refer to your memory aid, as needed.
  7. Solve.
  8. Ask yourself whether your answer makes sense.
  9. Write your answer.
- Do not leave a blank!

**My Strategies:**

The rule of the function is \_\_\_\_\_.



**Answer and Solution:**

The rule for a second degree function is the form  $f(x) = ax^2$

Use the coordinates (5, 10) in the function form to determine parameter "a".

$$f(x) = ax^2$$

$$(10) = a(5)^2$$

$$10 = 25a$$

$$\frac{10}{25} = \frac{25a}{25}$$

$$\frac{2}{5} \text{ or } 0.4 = a$$

$$f(x) = 0.4x^2$$

$$\text{or } f(x) = 0.4x^2$$

The rule of the function is  $f(x) = 0.4x^2$

**Specific Strategies:**

Recall the rule for the second degree function with vertex at the origin is  $y = ax^2$ .

Remember that in any algebraic equation, if you substitute known values, you can solve for the unknown remaining.

**ASK YOURSELF:**

What information is given in the graph?

An (x, y) point is given in the graph.

If you substitute x & y, then only parameter "a" will remain to be determined.

**Additional Resources:**

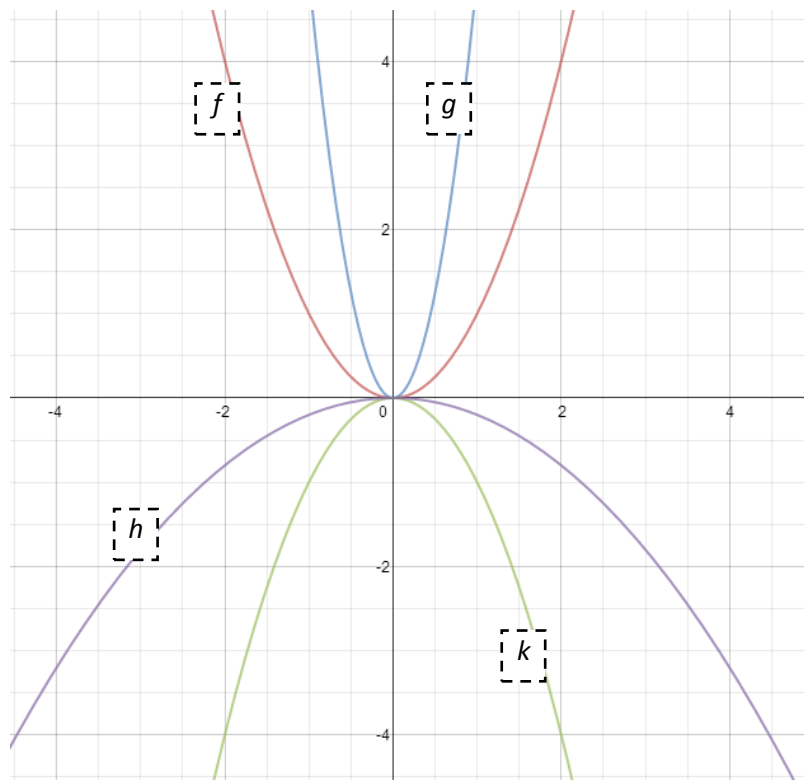
Visions Volume 2, pp. 28-29

Explore Learning Gizmos, <http://www.explorellearning.com/> look up:

- Quadratic Functions
- Quadratics in Polynomial Form- Activity A and Activity B (for these, you must keep the b and c sliders at position 0)

**Question:**

Match the second degree functions to their respective graphs.



Function	Graph
A) $y = 5x^2$	
B) $y = -0.2x^2$	
C) $y = x^2$	
D) $y = -x^2$	

**General Strategies:**

1. Read the question.
  2. Highlight key words.
  3. Identify the math topic.
  4. Re-read the question.
  5. Make a prediction about the answer- what will it look like? (an equation, a number, etc.).
  6. Refer to your memory aid, as needed.
  7. Solve.
  8. Ask yourself whether your answer makes sense.
  9. Write your answer.
- Do not leave a blank!

**My Strategies:**

**Answer and Solution:**

Function	Graph
E) $y = 5x^2$	<b><i>g</i></b>
F) $y = -0.2x^2$	<b><i>h</i></b>
G) $y = x^2$	<b><i>f</i></b>
H) $y = -x^2$	<b><i>k</i></b>

**Suggested Strategies:**

- Remember what effect parameter “a” has on the curve:
  - Positive opens up
  - Negative opens down
  - The larger the absolute value of “a” the narrower the curve

**Additional Resources:**

Visions Volume 2, Section 4.2, pp. 28-29

Explore Learning Gizmos, <http://www.explorelarning.com/> look up: Quadratics in Polynomial Form- Activity A and Activity B (for these, you must keep the b and c sliders at position 0)

**Question:**

Gordon is responsible for repairing the soccer field. He needs to purchase a square piece of turf that measures 22.5 m by 22.5 m. He finds the following 2 deals from two different companies:

**Company A:**

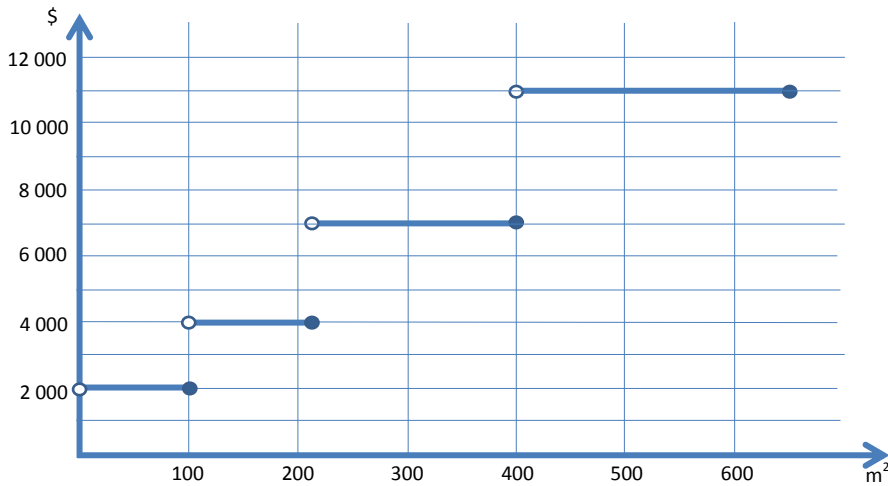
The turf is sold in square pieces and the price is calculated according to its area.

**Examples of Cost Based on the Rule Using Length of Side**

Side length of turf piece	Cost
10 m	\$ 1 800
17 m	\$ 5 202
25 m	\$11 250

**Company B:**

The turf is also sold by area but the pieces are not necessarily square.



Gordon will buy the piece of turf from the company with the lowest price.

How much will Gordon pay for the piece of turf?

**General Strategies:**

1. Read the problem.
2. Highlight key words.
3. Identify the math topics.
4. Re-read the problem.
5. Define your steps (your game plan) – this is criteria 3.
6. Refer to your memory aid, as needed.
7. Solve.
8. Ask yourself whether your answer makes sense.
9. Write your answer statement.

Show any or all your work! Do not leave a blank page!

**My Strategies:**

**Answer and Solution:****Company A:**

You can use any of the points to substitute them in  $y = ax^2$

$$y = ax^2$$

$$(1800) = a(10)^2$$

$$1800 = 100a$$

$$\frac{1800}{100} = \frac{100a}{100}$$

$$a = 18$$

$$\text{rule : } y = 18x^2$$

Let  $x$  be the length of side

Let  $y$  be the cost of turf

Now that you have the rule, you can substitute accordingly:

Turf piece of 22.5m ( $x$  value):

$$y = 18 \times (22.5)^2$$

$$y = 9112.5$$

$$\mathbf{\$9\ 112.50}$$

**Company B:**

Calculate the area of the square piece of turf:

$$22.5 \times 22.5 = 506.25\ m^2$$

Reading the graph, you can see that  $506.25\ m^2$  corresponds to a cost of **\$11 000**.

Gordon will buy the turf at the lowest price and therefore he will buy from Company A.

**Gordon will pay \$9 112.50 for the turf.**

**Suggested Strategies:****Company A:**

Turf is sold in SQUARE pieces and the cost depends on the AREA ( $s^2$ )

**ASK YOURSELF:**

Which kind of function is associated with squaring a number?  $y = ax^2$

Substitute ( $x, y$ ) value and solve for "a". This will give you your rule and you can then substitute and solve for  $y$ .

**Company B:**

Before you can use this graph, you need to calculate the area of the turf Gordon needs.

**Additional Resources:**

Visions Volume 2, pp. 28-29

Khan Academy video: [https://www.khanacademy.org/math/algebra/quadratics/solving\\_graphing\\_quadratics/v/graphing-a-quadratic-function](https://www.khanacademy.org/math/algebra/quadratics/solving_graphing_quadratics/v/graphing-a-quadratic-function)

<http://www.purplemath.com/modules/grphquad.htm>

Explore Learning Gizmos, <http://www.explorelearning.com/> look up: Quadratics in Polynomial Form- Activity A and Activity B (for these, you must keep the  $b$  and  $c$  sliders at position 0)



## 4.3 Exponential Function

**Question:**

A house, initially valued at \$275 000, increases in value by 2% annually.

Let:

$x$  : represent the number of years and

$f(x)$ : represent the value of the house,

Which of the following equations defines this situation?

A)  $f(x) = 275\,000(0.02)^x$

B)  $f(x) = 275\,000(1.02)^x$

C)  $f(x) = 275\,000(1.2)^x$

D)  $f(x) = 275\,000(0.98)^x$

**General Strategies:**

1. Read the question.
  2. Highlight key words.
  3. Identify the math topic.
  4. Re-read the question.
  5. Make a prediction about the answer- what will it look like? (an equation, a number, etc.).
  6. Refer to your memory aid, as needed.
  7. Solve.
  8. Ask yourself whether your answer makes sense.
  9. Write your answer.
- Do not leave a blank!

**My Strategies:**



**Answer and Solution:**

Exponential Function:

$$f(x) = a(c)^x$$

where:  $a$  is the initial value and  
 $c$  is the base

Given the value increases,  $C > 1$

$2/100 = 0.02$  and then adding it to 1

$$C = 100\% + 2\% = 102\% = 1.02$$

The correct equation is  $f(x) = 275\,000 (1.02)^x$

- A)  $f(x) = 275\,000 (0.02)^x$  This would be a decreasing function since the base is less than 1.
- B)  $f(x) = 275\,000 (1.02)^x$  This is correct.**
- C)  $f(x) = 275\,000 (1.2)^x$  This would represent a growth rate of 20% not 2%.
- D)  $f(x) = 275\,000(0.98)^x$  This would represent a decay of 2% (decreasing).

**The answer is B.**

**Suggested Strategies:**

- Find the type of function for the situation.
- Calculate “ $c$ ” by using the percentage.
- Remember that in growth situations we add to 100%.

**Additional Resources:**

Visions Volume 1, pp. 39-41

**Question:**

Given the exponential function  $f(x) = 505 (0.94)^x$ ,

Where,

$x$  : represent the numbers of years since 2010

$f(x)$  : represent the cost of the bike

Which of the following statements is true?

- A) The initial value is 0.94.
- B) The bike's value decreases by 94% yearly.
- C) The function is increasing.
- D) In the year 2020, the value of the bike will be \$272.

**General Strategies:**

1. Read the question.
  2. Highlight key words.
  3. Identify the math topic.
  4. Re-read the question.
  5. Make a prediction about the answer- what will it look like? (an equation, a number, etc.).
  6. Refer to your memory aid, as needed.
  7. Solve.
  8. Ask yourself whether your answer makes sense.
  9. Write your answer.
- Do not leave a blank!

**My Strategies:**

**Answer and Solution:**

The initial value is 505, not 0.94.

The bike's value decreases by 6%, not 94%.

This is a decreasing function because 0.94 is less than 1.

The value of the bike will be \$272 in the year 2020.

Proof:

2020 – 2010 = 10 years

$$f(10) = 505(0.94)^{10} = 272$$

- A) The initial value is 0.94.  
No – the initial value is \$505.
- B) The bike decreases by 94% yearly.  
No – the rate at which it decreases is  $1 - 0.94 = 0.06$ , or 6%
- C) The graph is an increasing function.  
No – since the base is less than one, it is a decreasing function.

**D) In the year 2000, the value of the bike will be \$272. Yes:**

2000 – 1990 = 10 years

$$f(10) = 505(0.94)^{10} = 272$$

**The answer is D.**

**Suggested Strategies:**

- Determine what the initial value is and compare with A).
- Determine the percentage decrease and compare with B).
- Determine if the function is increasing and compare with C).
- Calculate the number of years from 2010 to 2020 and replace value in x. Compare with D).

**Additional Resources:**

Visions Volume 1, pp. 39-41

**Question:**

The value of a video game depreciates 35% yearly. In 5 years, the price of the video game will be \$10.21.

What is the initial price of the video game?

The initial price of video game is \_\_\_\_\_.

**General Strategies:**

1. Read the question.
  2. Highlight key words.
  3. Identify the math topic.
  4. Re-read the question.
  5. Make a prediction about the answer- what will it look like? (an equation, a number, etc.).
  6. Refer to your memory aid, as needed.
  7. Solve.
  8. Ask yourself whether your answer makes sense.
  9. Write your answer.
- Do not leave a blank!

**My Strategies:**

**Answer and Solution:**

$$f(x) = ac^x$$

$$c = 1 - 0.35 = 0.65$$

$$f(x) = a(0.65)^x$$

$$10.21 = a(0.65)^5$$

$$\frac{10.21}{0.1160} = \frac{0.1160 a}{0.1160}$$

$$a = \$88.02$$

**Suggested Strategies:**

- Notice that this is an exponential function that is decreasing.
- Write the equation for this function.
- Calculate the value of “c” which uses the percentage.
- Fill in x with the number of years and y with the price after 5 years.
- Work backwards to find  $a$ , the initial price.
- Remember that in decay or decreasing situations we subtract from 100%.

**Answer:** The initial price of the video game is \$88.02.

**Additional Resources:**

Visions Volume 1, pp. 39-41

**Question:**

The function  $f$  described below represents the number of bacteria in a well, in relation to the amount of time elapsed since 2005.

$$f(x) = 4500(1.33)^x$$

where:

$x$  is the number of years elapsed since 2005

$f(x)$  is the number of bacteria

In what year will the number of bacteria be 137 858?

In \_\_\_\_\_ the number of bacteria will be 137 858.

**General Strategies:**

1. Read the question.
  2. Highlight key words.
  3. Identify the math topic.
  4. Re-read the question.
  5. Make a prediction about the answer- what will it look like? (an equation, a number, etc.).
  6. Refer to your memory aid, as needed.
  7. Solve.
  8. Ask yourself whether your answer makes sense.
  9. Write your answer.
- Do not leave a blank!

**My Strategies:**

**Answer and Solution:**

$$137\,858 = 4500(1.33)^x$$

**OR**

x	$4500(1.33)^x$	$f(x)$
1	$4500(1.33)^1$	5985.0
2	$4500(1.33)^2$	7960.1
3	$4500(1.33)^3$	10586.9
4	$4500(1.33)^4$	14080.5
5	$4500(1.33)^5$	18727.1
6	$4500(1.33)^6$	24907.1
7	$4500(1.33)^7$	33126.4
8	$4500(1.33)^8$	44058.1
9	$4500(1.33)^9$	58597.3
10	$4500(1.33)^{10}$	77934.4
11	$4500(1.33)^{11}$	103652.7
<b>12</b>	<b><math>4500(1.33)^{12}</math></b>	<b>137858.1</b>
13	$4500(1.33)^{13}$	183351.2

$x = 12$  years later

$$2005 + 12 = 2017$$

**Answer: In 2017 the number of bacteria will be 137 858.**

**Suggested Strategies:**

- Replace  $y$  by the number of bacteria given.
- Work backwards to find the value of  $x$  by guess and check.

**OR**

- Set up a table of values for the function and find the  $y$  value you are looking for.
- In the table, you can jump ahead to where you think  $x$  would work.
- Your table should contain more than two calculations in order to show evidence of your thinking.

**Additional Resources:**

Visions Volume 1, pp. 39-41

**Question:**

Sophia invested \$5000 today. Her investment will increase by 2.5% each year.

How much profit will she have made in 10 years?

Sophia will have made \_\_\_\_\_ profit in 10 years.

**General Strategies:**

1. Read the question.
  2. Highlight key words.
  3. Identify the math topic.
  4. Re-read the question.
  5. Make a prediction about the answer- what will it look like? (an equation, a number, etc.).
  6. Refer to your memory aid, as needed.
  7. Solve.
  8. Ask yourself whether your answer makes sense.
  9. Write your answer.
- Do not leave a blank!

**My Strategies:**



**Answer and Solution:**

$$f(x) = ac^x$$

$a = \text{initial value} = 5000$  (principal)

$c = \text{growth rate} = 1 + 0.025 = 1.025$

OR  $100\% + 2.5\% = 102.5\%$  or  $1.025$

$$f(x) = 5000 (1.025)^x$$

where:

$x$  is the number of years and

$f(x)$  is the total of principal and interest

$$f(10) = 5000 (1.025)^{10}$$

$$f(x) = \$6400.42$$

$$6400.42 - 5000 = \$1400.42$$

**Answer: Sophia will have made \$1 400.42 profit in 10 years.**

**Suggested Strategies:**

- Notice this is an exponential function and that it is increasing so the base is greater than 1.
- Write the equation associated with the function.
- Substitute the initial value for “ $a$ ”.
- Determine “ $c$ ” by using the percentage.
- Recognize that  $x$  represents the number of years and  $y$  represents the total amount.
- Plug in number of years for “ $x$ ”.

**Additional Resources:**

Visions Volume 1, pp. 39-41

**Question:**

Amy and Ben have deposited money in different banks.

Amy initially deposited \$400 in the bank, and deposits \$10 into her account every month. No interest is earned.

Ben made a one-time investment of \$850 at a yearly interest rate of 4%.

Who will have more money saved after 5 years?

\_\_\_\_\_ will have more money after 5 years.

**General Strategies:**

1. Read the question.
  2. Highlight key words.
  3. Identify the math topic.
  4. Re-read the question.
  5. Make a prediction about the answer- what will it look like? (an equation, a number, etc.).
  6. Refer to your memory aid, as needed.
  7. Solve.
  8. Ask yourself whether your answer makes sense.
  9. Write your answer.
- Do not leave a blank!

**My Strategies:**

**Answer and Solution:**

Amy:

$$f(x) = 10x + 400$$

where:

 $x$  is the number of months $f(x)$  is the total amount of money in the bankAfter 5 years ( $5 \times 12 = 60$  months), the total amount is:

$$f(60) = 10(60) + 400 = \$1000$$

Ben:

$$f(x) = 850(1.04)^x$$

where:

 $x$  is the number of years $f(x)$  is the total amount of money in the bank

After 5 years, the total amount is:

$$f(5) = 850(1.04)^5 = \$1034.15$$

The person who will have the most money after 5 years is Ben.

**Answer: Ben will have more money after 5 years.****Suggested Strategies:**

- Find the function representing Amy & Ben's situation.
- If the function is linear then " $a$ " is the amount placed monthly and " $b$ " is the initial amount invested.
- If the function is exponential then " $a$ " is the amount invested and " $c$ " is calculated by using the percentage given.
- Replace  $x$  in both equations by the amount of years given.

**Additional Resources:**

Visions Volume 1, pp. 39-41

**Question:**

A study examined the populations of four neighboring towns.

Town A

In 1960, Town A had 5000 inhabitants. Since then, there has been an equal amount of births as there have been deaths and the number of people moving away has matched the number of people moving to the town.

Town B

Function  $f$  described below represents the population of Town B in relation to the time elapsed since 2001.

$$f(x) = 2000 (1.022)^x$$

where:

$x$  represents time elapsed since 2001, in years

$f(x)$  represents the population of Town B

Town C

In 2010, Town C had 5000 inhabitants. The population has decreased by 50 inhabitants every year.

Town D

In 2006, Town D had a population of 1500. It is estimated that the population will increase by 5% annually.

The four towns will be merged in 2020 to form one city.

What will the population of the new city be when it is formed in 2020?

The population of the new city will be \_\_\_\_\_.

**General Strategies:**

1. Read the problem.
2. Highlight key words.
3. Identify the math topics.
4. Re-read the problem.
5. Define your steps (your game plan) – this is criteria 3.
6. Refer to your memory aid, as needed.
7. Solve.
8. If you get stuck on a calculation, pick a number and keep going.
9. Ask yourself whether your answer makes sense.
10. Write your answer statement.

Show any or all your work! Do not leave a blank page!

**My Strategies:**

**Answer and Solution:**

Town A: constant function

$$f(x) = 5000,$$

where:

$x$  is the number of years elapsed since 1960

$f(x)$  is the total inhabitants.

Town B: exponential function

2020 – 2001 = 19 years elapsed.

$$f(19) = 2000 (1.022)^{19} = 3024 \text{ inhabitants}$$

Town C: linear function

2020 – 2010 = 10 years elapsed

$$f(x) = 5000 - 50(x)$$

$$f(19) = 5000 - 50(19) = 4050 \text{ inhabitants.}$$

Town D : exponential function

For  $c = 100\% + 5\%$

$$= 105\% = 1.05$$

$a =$  initial population of 1500

$$f(x) = ac^x$$

$$f(x) = 1500 (1.05)^x$$

$$x = 2020 - 2006 = 14$$

$$f(14) = 1500 (1.05)^{14} = 2969 \text{ inhabitants}$$

Total:

$$5000 + 3024 + 4050 + 2969 = 15\,043 \text{ inhabitants}$$

**Answer: The population of the new city will be 15 043.**

**Suggested Strategies:**

- Determine the type of function for each town and write the equation.
- Determine the number of years that pass between the time of the merging and the creation of each town.
- Replace  $x$  by the number of years that pass in each equation.

**Additional Resources:**

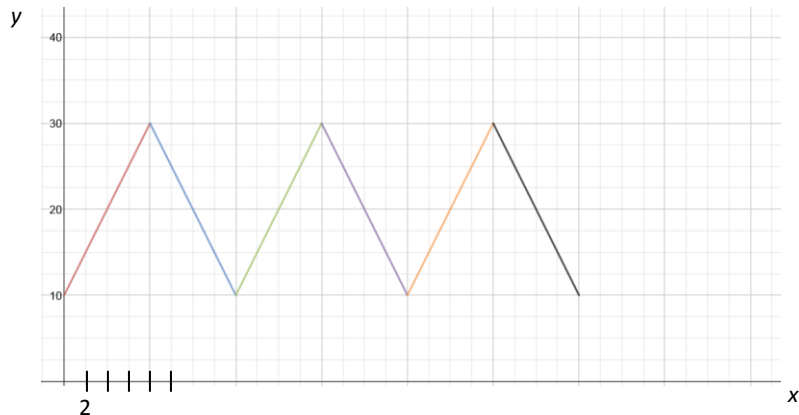
Visions Volume 1, pp. 39-41



## **4.4 Step, Periodic and Piecewise Functions**

**Question:**

The following graph represents a periodic function.



What is the period of this function?

- A) 8
- B) 16
- C) 24
- D) 48

**General Strategies:**

1. Read the question.
2. Highlight key words.
3. Identify the math topic.
4. Re-read the question.
5. Refer to your memory aid, as needed.
6. Solve the problem without looking at choices shown (A, B, C and D).
7. Look at all the choices.
8. Match your answer to the appropriate choice.

Do not leave a blank!  
Make a choice!

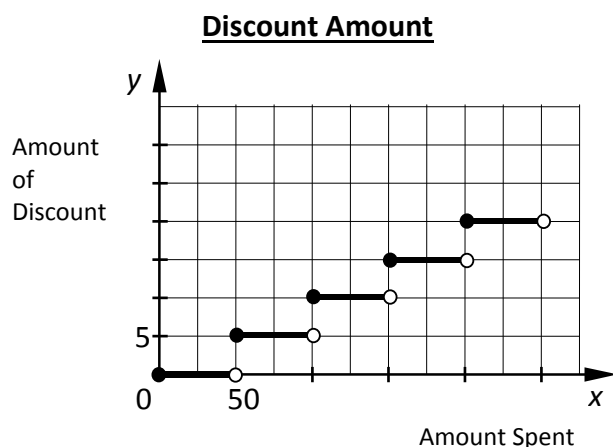
**My Strategies:**



<p><b>Answer and Solution:</b></p> <p>A) 8      This is the answer you get if you use a scale of 1 instead of 2.</p> <p><b>B) 16      Correct</b></p> <p>C) 24      This is the answer you get if you use the whole graph and not just one period, and a scale of 1 instead of 2.</p> <p>D) 48      This is the answer you get if you use the whole graph and not just one period.</p> <p><b>The answer is B.</b></p>	<p><b>Suggested Strategies:</b></p> <p>To find the period of a periodic function means that the <math>y</math> values will repeat over a certain interval of <math>x</math>.</p> <p>The period can also be thought of as the length of one cycle.</p> <p>In this case, the graph starts at 10, goes to 30, and back down to 10. At this point the graph starts repeating. Look at the <math>x</math> values to find the period. In this case the graph repeats after 8 units/squares but each unit is worth 2 so the period is 16 units.</p>
<p><b>Additional Resources:</b></p> <p>Visions Volume 2, Section 4.4, p. 53</p>	

**Question:**

A store offers a discount of \$5 for every \$50 in purchases. The graph below illustrates the relation between the value of the purchases and the amount of discount a customer receives.



Consider the following five statements regarding the graph.

1. A customer who spends \$150 will receive a \$10 discount.
2. A customer who spends \$75 will receive a \$5 discount.
3. A customer will receive a \$5 discount when spending less than \$100.
4. A customer will receive twice as much of a discount when spending \$200 than \$100.
5. A customer will receive no discount when spending less than \$50.

Which of the statements above are true?

- A) 2, 4 and 5
- B) 2, 3 and 4
- C) 1, 2 and 4
- D) 1, 2 and 3

**General Strategies:**

1. Read the question.
2. Highlight key words.
3. Identify the math topic.
4. Re-read the question.
5. Refer to your memory aid, as needed.
6. Solve the problem without looking at choices shown (A, B, C and D).
7. Look at all the choices.
8. Match your answer to the appropriate choice.

Do not leave a blank!  
Make a choice!

**My Strategies:**

**Answer and Solution:**

Read each statement carefully.

1. A customer that spends \$150 will receive a \$10 discount.

*False—\$150=\$15 discount*

2. A customer that spends \$75 will receive a \$5 discount.

*True – In values between the endpoints of the step has the same y-value.*

3. A customer will receive a \$5 discount when spending less than \$100.

*False – Be careful with the less than \$100. Less than \$100 includes less than \$50. Less than \$50 equals \$0 discount.*

4. A customer will receive twice as much of a discount when spending \$200 than \$100.

*True – The discount for \$200 = \$20 and \$100= \$10*

5. A customer will receive no discount when spending less than \$50.

*True – Less than \$50 is the step on the x-axis.*

**The answer is A.**

**Suggested Strategies:**

1) “*greatest integer function*” is another name for *step function*.

2) Read each statement carefully and determine if it is true or false.

3) When reading the endpoints of the steps know the difference between an open circle and filled in circle.



Endpoints

- A closed point means it is included.
- An open point means it is not included.

4) The question is looking for the true statements. Sometimes the question wants the *false* statements. Take a moment to highlight the word true and indicate each statement as true or false as you read them.

**Additional Resources:**

Visions Volume 2, Section 4.4, p. 53

**Question:**

The cost to park a car in a particularly expensive lot is \$40 for the first half hour and \$5.00 for each additional hour or part thereof.

A customer uses this parking lot for five hours.

How much will the customer pay for parking?

The customer will pay \$\_\_\_\_\_ for parking 5 hours.

**General Strategies:**

1. Read the question.
  2. Highlight key words.
  3. Identify the math topic.
  4. Re-read the question.
  5. Make a prediction about the answer- what will it look like? (an equation, a number, etc.).
  6. Refer to your memory aid, as needed.
  7. Solve.
  8. Ask yourself whether your answer makes sense.
  9. Write your answer.
- Do not leave a blank!

**My Strategies:**

**Answer and Solution:**

Making a table to show how the cost changes over time.

Cost to Park a Car

Time (hours)	Total Cost
]0,0.5]	\$40
]0.5,1.5]	\$45
]1.5,2.5]	\$50
]2.5,3.5]	\$55
]3.5,4.5]	\$60
]4.5,5.5]	\$65
]5.5,6.5]	\$70

ANSWER

The question asks for 5 hours which is located in the interval ]4.5,5.5] at a cost of \$65.

Notice the use of intervals. The interval ]0, 0.5] indicates the hours ranging from anything above 0 up to and including 0.5 hours. Although the customer parks a car for  $\frac{1}{4}$  of an hour, they will be charged \$40.

OR

You could also consider the very specific case: \$40 for the first half hour plus 4.5 hours → rounded up to 5 hours x \$5 per hour:  $40 + (5 \times 5) = 40 + 25 = 65$ .

**Answer: The customer will pay \$65 for parking 5 hours.**

**Suggested Strategies:**

- 1) Identify that this problem is a step function.
- 2) Note the key words in the problem. “for each additional hour or part thereof.”

Take a moment to highlight the key words.

“part thereof” means that the customer will be charged for \$5 even if they did not complete one hour.

- 3) Recall the use of intervals. Notice the difference in the brackets. ]0, 0.5]

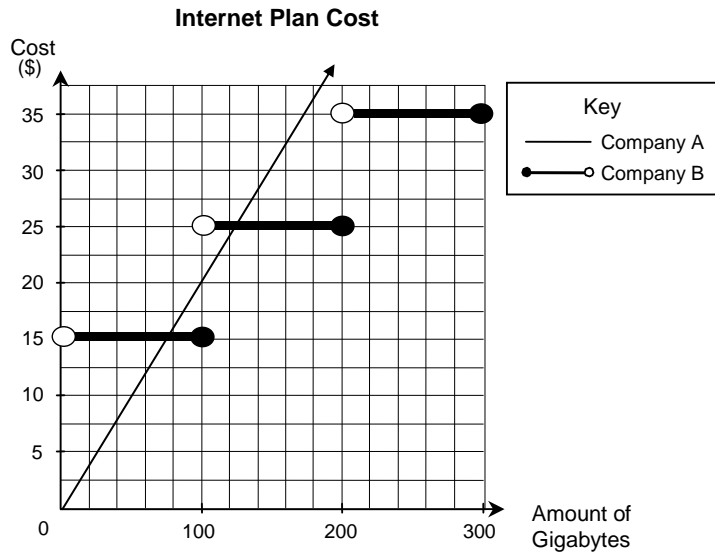
The bracket here means including 0.5, while the other bracket does not include 0.

**Additional Resources:**

Visions Volume 2, Section 4.4, p. 53

**Question:**

Two companies offer different prices for internet service. Company A uses a linear model where each 100 gigabytes of usage will cost \$20. Company B follows a greatest integer function as shown on the graph below.



What is the difference in cost between the two companies for 200 gigabytes?

The difference in cost is \_\_\_\_\_.

**General Strategies:**

1. Read the question.
2. Highlight key words.
3. Identify the math topic.
4. Re-read the question.
5. Make a prediction about the answer- what will it look like? (an equation, a number, etc.).
6. Refer to your memory aid, as needed.
7. Solve.
8. Ask yourself whether your answer makes sense.
9. Write your answer.

Do not leave a blank!

**My Strategies:**

**Answer and Solution:****Company A:**

$$\frac{100 \text{ gb}}{\$20} = \frac{200 \text{ gb}}{\$x} \rightarrow x = \$40$$

**OR**

We can see the line passing through the origin (0, 0) and the given point (100, 20)

$$\begin{aligned} &\$20 \text{ per } 100 \text{ gigabytes} \\ &\$0.20 \text{ per } 1 \text{ gigabyte} \end{aligned}$$

The function rule to calculate the cost in relation to the internet usage (gigabyte) will be  $f(x) = 0.20x$

The cost of 200 gigabyte  $f(200) = \$40$

**Company B:**

From the graph –  $f(200) = \$25$

(Note that it isn't \$35 because it's the solid dot that indicates the y-value.)

The difference:  $\$40 - \$25 = \$15$

**Answer: The difference in cost is \$15.**

**Suggested Strategies:**

- 1) Since we can't read off the graph for the cost of 200 gb from Company A, we need to look further.
  - The text says each 100 gb costs \$20 so we could set up a proportion or
  - The graph shows the same information but we can find a unit rate which will be the "a" in  $y = ax + b$ .  $b$  will be 0 since the line goes through the origin.

Recall: 'a' in the equation represents the slope  $\frac{y_2 - y_1}{x_2 - x_1}$  and 'b' represent the y-intercept. (In this case it is 0 since the line passes through the origin.)

- 2) By looking at the graph we can see the exact value of the cost of Company B at 200 gb. Be careful not to take the \$35 value. Remember the difference between a white circle and a black circle.

**Additional Resources:**

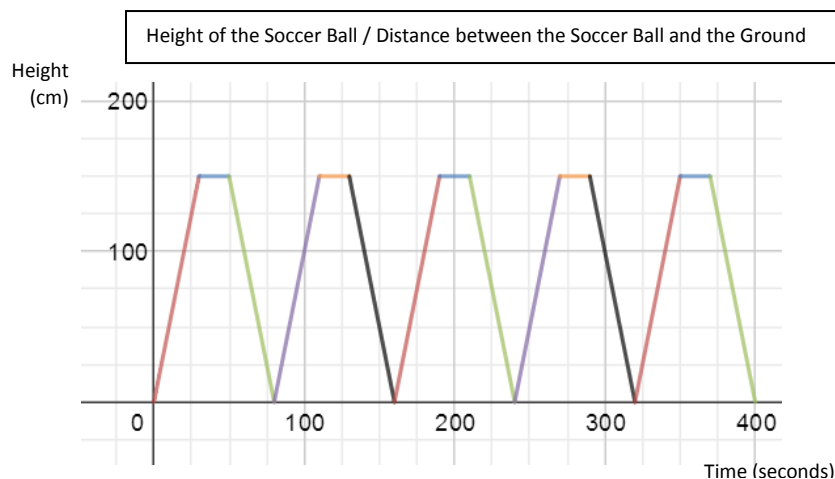
Visions Volume 2, Section 4.4, p. 53

**Question:**

A store selling World Cup memorabilia places a mechanical mascot in front of the store.

The mascot raises a ball from the ground to a maximum height of 150 cm at a constant rate, holds it there for 20 seconds, and then lowers it back to ground level at the same rate.

The graph below illustrates a periodic function that represents the height, or the distance between the ball and the ground in relation to the time elapsed in seconds.



A store employee turns on the mechanism that moves the soccer ball at 8:00AM. At that point the ball is at ground level. At exactly 8:15 AM, the mechanism breaks down and the soccer ball stops moving.

How high above the ground is the ball when the mascot stops moving?

The ball is \_\_\_\_\_ cm off the ground when the mascot stops moving.

**General Strategies:**

1. Read the problem.
2. Highlight key words.
3. Identify the math topics.
4. Re-read the problem.
5. Define your steps (your game plan) – this is criteria 3.
6. Refer to your memory aid, as needed.
7. Solve.
8. If you get stuck on a calculation, pick a number and keep going.
9. Ask yourself whether your answer makes sense.
10. Write your answer statement.

Show any or all your work! Do not leave a blank page!

**My Strategies:**



**Answer and Solution:**

**Step 1: Determine the period of the function**

5 complete cycles = 400 seconds    1 complete cycle = 80 seconds

**Step 2: Determine how many complete cycles from 8:00-8:15 AM**

Since our unit of time on our graph is in seconds we need to convert minutes to seconds.

15 minutes x 60 seconds/minute = 900 seconds

Determine the number of complete cycles in 900 seconds

$$\frac{900 \text{ seconds}}{80 \frac{\text{seconds}}{\text{cycle}}} = 11.25 \text{ cycles}$$

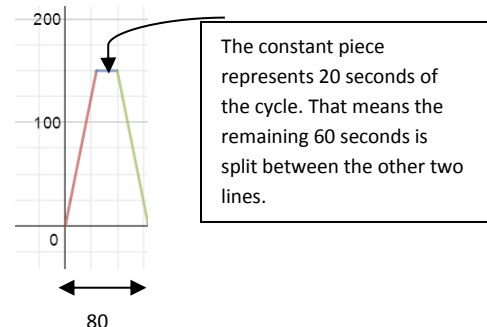
11 Full Cycles means that the ball will return back to where it started (i.e. ground)  
0.25 of a cycle represents quarter of a full cycle. (0.25 x 80seconds = 20 seconds)

**OR**

The graph (5 cycles) covers 400 seconds, so 800 seconds covers the graph twice, leaving 100 seconds (900 – 800) left over to being the third time. From reading the graph, you can see that at 100 seconds, the ball is 100 cm above the ground. HOWEVER, even though it *looks* like 100, make sure by showing the following:

**Step 3: Determine the height the ball is relative to the ground at 20 seconds**

To find the exact value we need to break a cycle into pieces.



At 20 seconds the ball will be moving upwards. To know the exact height we will have to find the equation of a line:  $y = ax + b$ .

We have two points on the line.  $P_1(0,0)$  and  $P_2(25,125)$

$$a = \frac{y_2 - y_1}{x_2 - x_1} = \frac{125 - 0}{25 - 0} = \frac{125}{25} = 5 \text{ and } b = 0 \text{ since the } y\text{-intercept is at the origin.}$$

The equation of the line is  $y = 5x$

To find the height of the ball at 20 seconds, substitute  $x=20$  and solve for  $y$ .

$$y = 5(20) = 100 \text{ cm}$$

**The ball is 100 cm off the ground when the mascot stops moving.**

**Suggested Strategies:**

- 1) Determine the period (the length of a full cycle) of the periodic function.

*Note: The scale of x-axis is  $100/4 = 25$  seconds per grid mark.*

In this case you cannot determine the exact value of the period from looking at one cycle on the graph. Instead we can see that 5 full cycles equals 400 seconds.  
(1 cycle = 80seconds)

- 2) At 8:00 AM the ball starts at ground level and moves for 15 minutes. We need to figure out how many complete cycles we have completed in 15 minutes and see what's left over.

- 3) Write the equation of a line given two points.  
 $y = ax + b$

Recall: 'a' in the equation represents the slope  $\frac{y_2 - y_1}{x_2 - x_1}$  and 'b' represent the y-intercept.

**Additional Resources:**

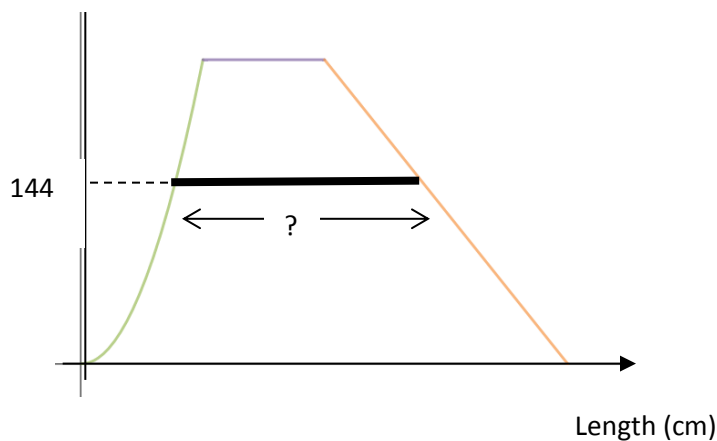
Visions Volume 2, Section 4.4, p. 53

**Question:**

The graph below represents the outline of a skateboard ramp which corresponds to a piecewise function defined by:

$$f(x) = \begin{cases} ax^2 & \text{if } 0 \leq x \leq 80 \\ 256 & \text{if } 80 \leq x \leq 160 \\ -1.25x + b & \text{if } 160 \leq x \leq 320 \end{cases}$$

Height (cm)



For security purposes, a strip of reflective tape will be placed on the ramp at a height of 144 cm.

**What is the length of this piece of reflective tape?**

The length of the piece of reflective tape is \_\_\_\_\_ cm.

**General Strategies:**

1. Read the problem.
2. Highlight key words.
3. Identify the math topics.
4. Re-read the problem.
5. Define your steps (your game plan) – this is criteria 3.
6. Refer to your memory aid, as needed.
7. Solve.
8. If you get stuck on a calculation, pick a number and keep going.
9. Ask yourself whether your answer makes sense.
10. Write your answer statement.

Show any or all your work! Do not leave a blank page!

**My Strategies:**

**Answer and Solution:****Step 1: Solving for the parameter “a” in the second-degree function**

Note: The second-degree function is connected to the constant function. To determine the coordinates of the point connecting the two functions we need to look at the interval in the function rule.

When  $x = 80$ , the  $y$ -coordinate is at 256.

Substitute the coordinates into the second-degree equation to solve for ‘a’

$$a(80)^2 = 256$$

$$a = \frac{256}{6400} = \frac{1}{25} \text{ or } 0.04$$

**Step 2: Solving for parameter “b” in the linear equation.**

The function connecting the linear function is the constant function.

When  $x = 160$ ,  $y = 256$

$$256 = -1.25(160) + b$$

$$256 + 200 = b$$

$$b = 456$$

**Step 3: Find the x coordinates when  $y = 144$** 

$\ln f(x) = 0.04x^2$	$\ln f(x) = -1.25x + 456$
$144 = 0.04x^2$	$144 = -1.25x + 456$
$3600 = x^2$	$-312 = -1.25x$
$x = 60$	$x = 249.6$

**Step 4: The length of tape**

$$249.6 \text{ cm} - 60 \text{ cm} = 189.6 \text{ cm}$$

The length of the piece of reflective tape is 189.6 cm

**Suggested Strategies:**

- 1) Identify the type of functions in the piecewise function.  
  
Remember a piecewise function is made up of different functions defined by a certain interval of  $x$ -values (domain).
- 2) To solve for the missing parameters you will need to know the coordinates of the points that connect the functions together
- 3) Find the  $x$  coordinates of the two extremities of the tape

**Additional Resources:**

Visions Volume 2 Section 4.4 p. 53

<http://www.mathsisfun.com/sets/functions-piecewise.html>

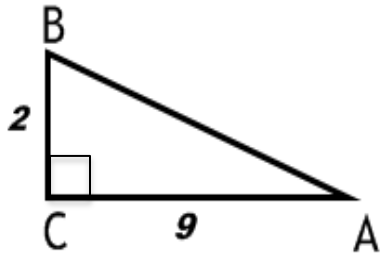
<http://www.purplemath.com/modules/strtlneq.htm>



# 5.1 Trigonometric Ratios

**Question:**

Consider the right triangle ABC shown below.



Which of the following expressions represents the correct trigonometric ratio for angle A?

A)  $\sin A = \frac{9}{2}$

B)  $\tan A = \frac{9}{2}$

C)  $\cos A = \frac{2}{9}$

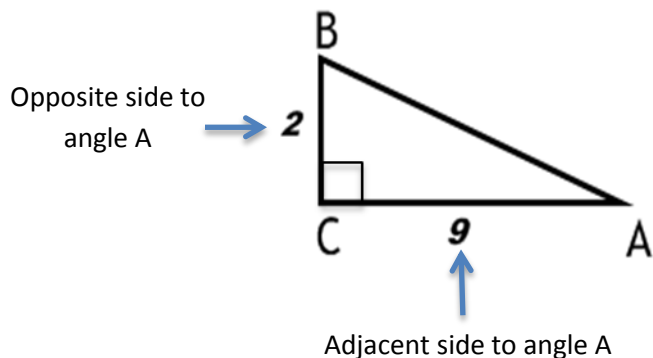
D)  $\tan A = \frac{2}{9}$

**General Strategies:**

1. Read the question.
2. Highlight key words.
3. Identify the math topic.
4. Re-read the question.
5. Refer to your memory aid, as needed.
6. Solve the problem without looking at choices shown (A, B, C and D).
7. Look at all the choices.
8. Match your answer to the appropriate choice.

Do not leave a blank!  
Make a choice!

**My Strategies:**

**Answer and Solution:**

Since the opposite and adjacent sides are given, the correct trigonometric ratio to select is Tangent (tan).

Tangent is the ratio of  $\frac{\text{opposite side of an angle}}{\text{adjacent side of an angle}}$

$$\text{So, } \tan A = \frac{2}{9}$$

- A)  $\frac{9}{2}$  would be cot  $A$  not sin  $A$
- B)  $\frac{9}{2}$  would be cot  $A$  not tan  $A$
- C)  $\frac{2}{9}$  would be tan  $A$  not cos  $A$
- D) **correct**

The answer is D.

**Specific Strategies:**

Which two sides are given in relation to angle  $A$ ?

OPPOSITE & ADJACENT

1. Identify the sides according to angle  $A$ .
2. Given the “knowns” (opposite and adjacent), select the correct trigonometric ratio.

**Additional Resources:**

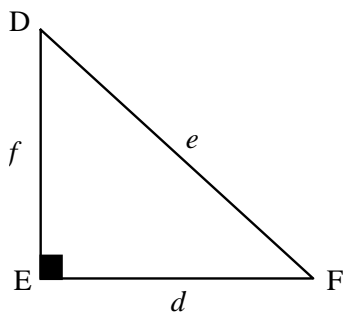
Visions Volume 2, p. 84

Khan Academy Video: <https://www.khanacademy.org/math/trigonometry/basic-trigonometry/basic-trig-ratios/v/basic-trigonometry>

<http://www.purplemath.com/modules/basirati.htm>

**Question:**

Consider right triangle DEF shown below.



Which trigonometric ratio corresponds to the ratio  $\frac{d}{e}$ ?

- A)  $\cos D$
- B)  $\tan D$
- C)  $\cos F$
- D)  $\sin F$

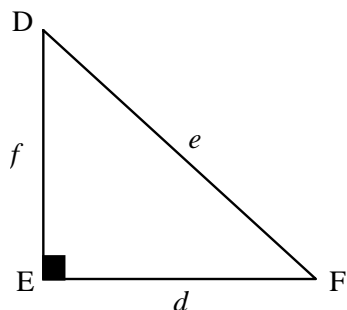
**General Strategies:**

1. Read the question.
2. Highlight key words.
3. Identify the math topic.
4. Re-read the question.
5. Refer to your memory aid, as needed.
6. Solve the problem without looking at choices shown (A, B, C and D).
7. Look at all the choices.
8. Match your answer to the appropriate choice.

Do not leave a blank!  
Make a choice!

**My Strategies:**



**Answer and Solution:**

A)  $\cos D = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{f}{e}$  incorrect

B)  $\tan D = \frac{\text{opposite}}{\text{adjacent}} = \frac{d}{f}$  incorrect

C)  $\cos F = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{d}{e}$  **correct**

D)  $\sin F = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{f}{e}$  incorrect

The answer is C.

**Specific Strategies:**

1. Consider each ratio one at a time
2. Label sides according to angle D
3. Label sides according to angle F

Always solve for each choice (A, B, C and D).

Don't stop when you think you have the correct answer.

**Additional Resources:**

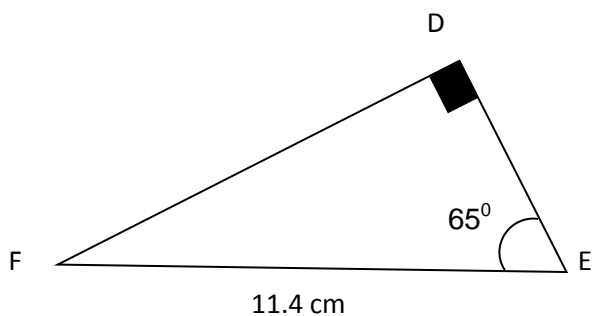
Visions Volume 2, p. 84

Khan Academy video: [https://www.khanacademy.org/math/trigonometry/basic-trigonometry/basic\\_trig\\_ratios/v/basic-trigonometry](https://www.khanacademy.org/math/trigonometry/basic-trigonometry/basic_trig_ratios/v/basic-trigonometry)

<http://www.purplemath.com/modules/basirati.htm>

**Question:**

Consider right triangle DEF below.



To the nearest hundredth, what is the length of side DF?

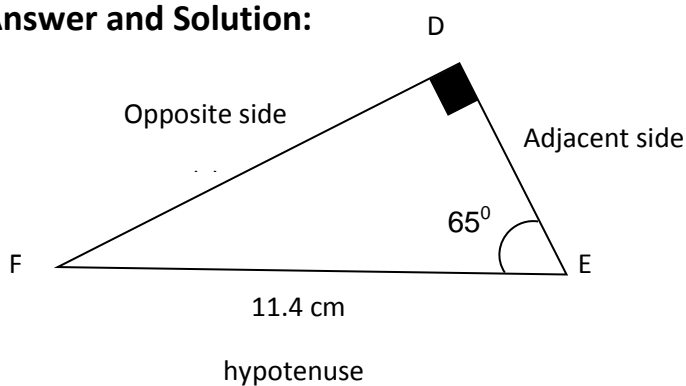
- A) 4.82 cm
- B) 10.33 cm
- C) 12.58 cm
- D) 24.45 cm

**General Strategies:**

1. Read the question.
2. Highlight key words.
3. Identify the math topic.
4. Re-read the question.
5. Refer to your memory aid, as needed.
6. Solve the problem without looking at choices shown (A, B, C and D).
7. Look at all the choices.
8. Match your answer to the appropriate choice.

Do not leave a blank!  
Make a choice!

**My Strategies:**

**Answer and Solution:**

- Length DF is the opposite side and the unknown ( $x$ )
- The hypotenuse is given
- Therefore, the correct trigonometric ratio to use is SIN (SOH)

**MAKE SURE YOUR CALCULATOR IS IN DEGREES**

$$\sin E = \frac{\textit{opposite}}{\textit{hypotenuse}}$$

$$\sin 65 = \frac{x}{11.4}$$

$$x = 11.4 \sin (65^\circ)$$

$$x = 10.33$$

- A) This is the result of  $\cos 65^\circ = \frac{x}{11.4}$  which is incorrect.
- B) **10.33 cm is correct.**
- C) This is the result of  $\sin 65^\circ = \frac{11.4}{x}$  which is incorrect.
- D) This is the result of  $\tan 65^\circ = \frac{x}{11.4}$  which is incorrect.

The answer is B.

**Specific Strategies:**

Which angle is given?

Which side is given?

Which side is the unknown?

Looking for an unknown in a right triangle means that you will need to use trigonometric ratios.

Remember: SOH CAH TOA

1. Label sides according to the given angle.
2. Select the appropriate trigonometric ratio (sin, cos or tan).
3. Solve for the unknown.

**Additional Resources:**

Visions Volume 2, p. 84

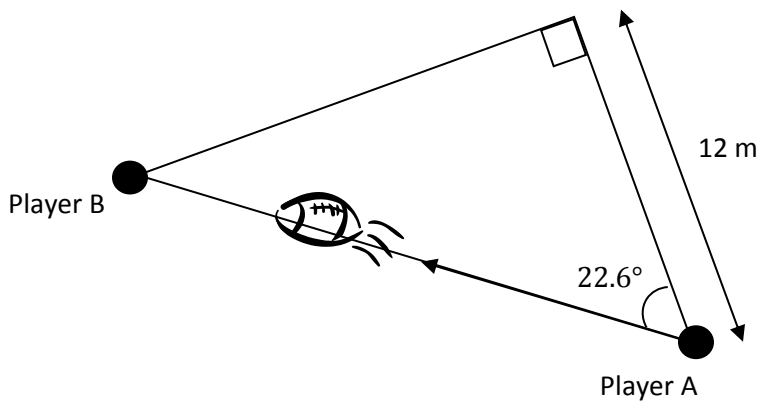
Khan Academy video: [https://www.khanacademy.org/math/trigonometry/basic-trigonometry/basic\\_trig\\_ratios/v/example-trig-to-solve-the-sides-and-angles-of-a-right-triangle](https://www.khanacademy.org/math/trigonometry/basic-trigonometry/basic_trig_ratios/v/example-trig-to-solve-the-sides-and-angles-of-a-right-triangle)

Explore Learning Gizmos, <http://www.explorelearning.com/> look up: Sine, Cosine, and Tangent

**Question:**

During a football practice, player A passes the ball to player B. The diagram below illustrates the pass.

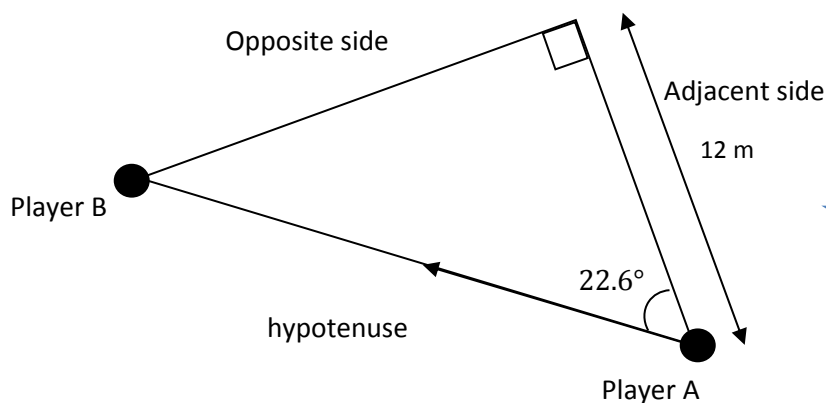
What is the distance traveled by the ball during the pass?

**General Strategies:**

1. Read the question.
  2. Highlight key words.
  3. Identify the math topic.
  4. Re-read the question.
  5. Make a prediction about the answer- what will it look like? (an equation, a number, etc.).
  6. Refer to your memory aid, as needed.
  7. Solve.
  8. Ask yourself whether your answer makes sense.
  9. Write your answer.
- Do not leave a blank!

**My Strategies:**

The distance traveled by the ball during the pass is \_\_\_\_\_ m.

**Answer and Solution:**

- Length of the adjacent side is given: 12 m
- The hypotenuse is the unknown ( $x$ )
- Therefore, the correct trigonometric ratio to use is COS (CAH)

**MAKE SURE YOUR CALCULATOR IS IN DEGREES**

$$\cos 22.6^\circ = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$0.9232 = \frac{12}{x}$$

$$x = \frac{12}{0.9232} = 12.998 \text{ or } 13 \text{ m}$$

**Answer:** The distance traveled by the ball during the pass is 13.0 m.

**Specific Strategies:**

Which angle is given?

Which side is given?

Which side is the unknown?

Looking for an unknown in a right triangle means that you will need to use trigonometric ratios.

Remember: SOH CAH TOA

1. Label sides according to the given angle.
2. Select the appropriate trigonometric ratio (sin, cos or tan).
3. Solve for the unknown.

**Additional Resources:**

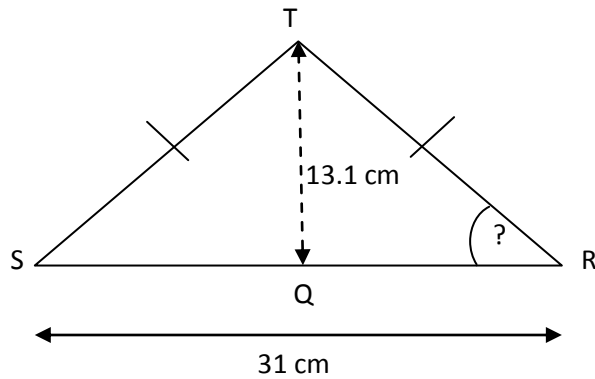
Visions Volume 2, p. 84; examples on p. 85

Khan Academy video: [https://www.khanacademy.org/math/trigonometry/basic-trigonometry/basic\\_trig\\_ratios/v/example--trig-to-solve-the-sides-and-angles-of-a-right-triangle](https://www.khanacademy.org/math/trigonometry/basic-trigonometry/basic_trig_ratios/v/example--trig-to-solve-the-sides-and-angles-of-a-right-triangle)

<http://www.purplemath.com/modules/basirati2.htm>

**Question:**

In isosceles triangle RST, height QT measures 13.1 cm.



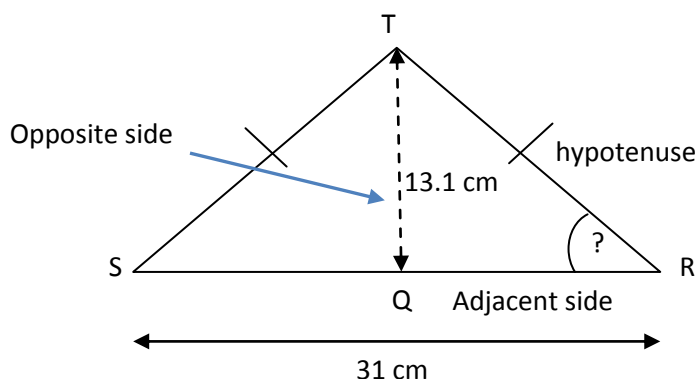
What is the measure of angle R to the nearest tenth?

The measure of angle R is \_\_\_\_\_.

**General Strategies:**

1. Read the question.
  2. Highlight key words.
  3. Identify the math topic.
  4. Re-read the question.
  5. Make a prediction about the answer- what will it look like? (an equation, a number, etc.).
  6. Refer to your memory aid, as needed.
  7. Solve.
  8. Ask yourself whether your answer makes sense.
  9. Write your answer.
- Do not leave a blank!

**My Strategies:**

**Answer and Solution:**

- The focus is on triangle TQR
- Triangle TQR is a right triangle because the height is perpendicular to the base.
- The measurement of QR is  $31 \div 2$  because the triangle is isosceles
- Use trigonometric ratios
- The “knowns” are the opposite side and adjacent side to angle R
- The correct trigonometric ratio is Tan (TOA)

Remember to divide RS by two to obtain the value of the adjacent side of angle R.  $31 \div 2 = 15.5$

$$\tan R = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan R = \frac{13.1}{15.5}$$

$$R = \tan^{-1} 0.8451$$

$$R = 40.2^\circ$$

The measure of angle R is  $40.2^\circ$ .

**Specific Strategies:**

Which angle is given? or unknown?

Which sides are given?

Looking for an unknown in a right triangle means that you will need to use trigonometric ratios.

Remember: SOH CAH TOA

1. Label sides according to the given angle

2. Select the appropriate trigonometric ratio (sin, cos or tan)

3. Solve for the unknown

**Additional Resources:**

Visions Volume 2, pp. 84-85

Khan Academy video: [https://www.khanacademy.org/math/trigonometry/basic-trigonometry/basic\\_trig\\_ratios/v/example--trig-to-solve-the-sides-and-angles-of-a-right-triangle](https://www.khanacademy.org/math/trigonometry/basic-trigonometry/basic_trig_ratios/v/example--trig-to-solve-the-sides-and-angles-of-a-right-triangle)  
<http://www.purplemath.com/modules/basirati2.htm>

**Question:**

In the figure below, triangle BEF and triangle ABC are right triangles.

In addition,

$\overline{DB}$  is an altitude of right triangle ABC

$$\overline{AE} \cong \overline{BE}$$

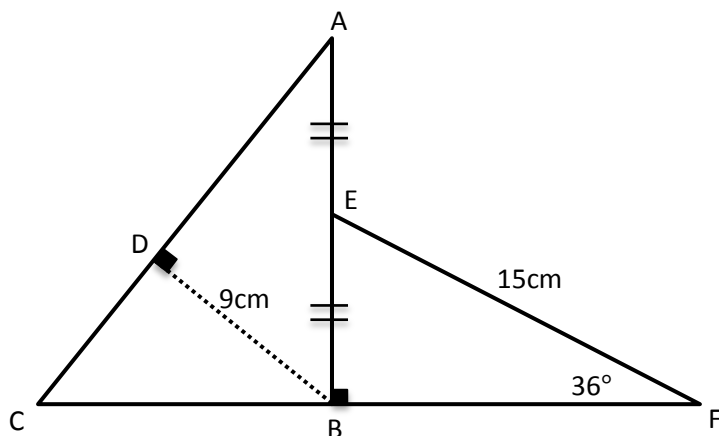
$$m\angle BEF = 36^\circ$$

$$m\overline{EF} = 15\text{cm}$$

$$m\overline{DB} = 9\text{cm}$$

The drawing is not to scale.

What is the area of figure AEFC?



The area of figure AEFC is \_\_\_\_\_  $\text{cm}^2$ .

**General Strategies:**

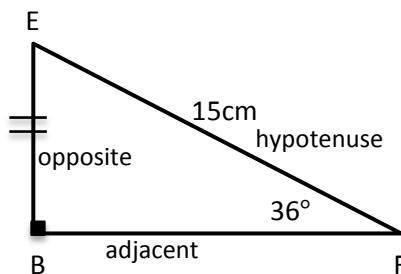
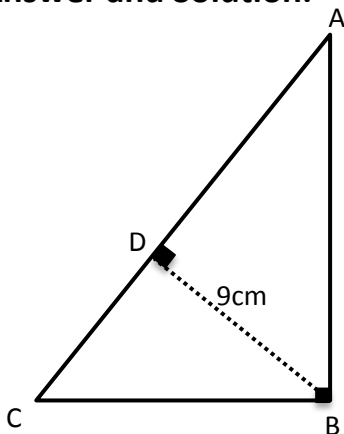
1. Read the problem.
2. Highlight key words.
3. Identify the math topics.
4. Re-read the problem.
5. Define your steps (your game plan) – this is criteria 3.
6. Refer to your memory aid, as needed.
7. Solve.
8. If you get stuck on a calculation, pick a number and keep going.
9. Ask yourself whether your answer makes sense.
10. Write your answer statement.

Show any or all your work! Do not leave a blank page!

**My Strategies:**



**Answer and Solution:**



$$\begin{aligned} \overline{AB} &= 2 \times \overline{EB} \\ &= 2 \times 8.82 \\ &= 17.64\text{cm} \end{aligned}$$

$\overline{BD}$  is an altitude of triangle ABC, so we can use the Pythagorean theorem to find  $\overline{AD}$ :

$$\begin{aligned} \overline{AD}^2 + 9^2 &= 17.64^2 \\ \overline{AD} &= 15.17\text{cm} \end{aligned}$$

We can also use metric relations:

$$\begin{aligned} \overline{AB}^2 &= \overline{AD} \times \overline{AC} \\ 17.64^2 &= 15.17 \times \overline{AC} \\ \overline{AC} &= 20.51\text{ cm} \end{aligned}$$

$$\begin{aligned} \overline{AC} \times \overline{DB} &= \overline{AB} \times \overline{BC} \\ 20.51 \times 9 &= 17.64 \times \overline{BC} \\ \overline{BC} &= 10.46\text{cm} \end{aligned}$$

$$\text{Area} = \frac{\text{base} \times \text{height}}{2}$$

$$\begin{aligned} &= \frac{10.46 \times 17.64}{2} = \\ &91.32\text{ cm}^2 \end{aligned}$$

**Start** with triangle BEF:

Using the sin ratio, determine the length of side BE:

$$\sin(36^\circ) = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{\overline{BE}}{15}$$

$$15 \times 0.5878 \approx 8.82\text{cm}$$

Using the cos ratio (or Pythagorean Theorem), determine the length of side BF:

$$\cos(36^\circ) = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{\overline{BF}}{15}$$

$$15 \times 0.8090 \approx 12.14\text{cm}$$

Determine the area of triangle BEF:

$$\begin{aligned} \text{Area} &= \frac{\text{base} \times \text{height}}{2} \\ &= \frac{12.14 \times 8.82}{2} = 53.54\text{ cm}^2 \end{aligned}$$

**Specific Strategies:**

Break the image down into two right triangles: ABC and BEF. Determine the area of each and then add them.

$$\text{Area} = \frac{\text{base} \times \text{height}}{2}$$

Formulas that may apply to right triangles:

- Pythagorean theorem
- Trigonometric ratios
- Metric relations

$$\text{Total area} = 53.54\text{cm}^2 + 91.32\text{cm}^2 = \mathbf{144.86\text{ cm}^2}$$

**Additional Resources**

Visions Volume \_\_\_\_, Section 3.3, pp. 178-185

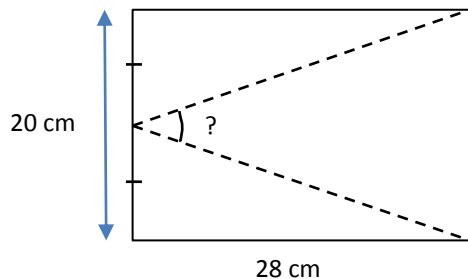
Visions Volume \_\_\_\_, Section 5.1, pp. 91-101



## **5.2 Finding Missing Measurements**

**Question:**

A sheet of construction paper is cut along the dashed lines.



What is the angle formed between the two dashed lines?

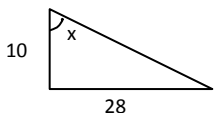
- A)  $19.7^\circ$
- B)  $39.4^\circ$
- C)  $70.3^\circ$
- D)  $140.6^\circ$

**General Strategies:**

1. Read the question.
2. Highlight key words.
3. Identify the math topic.
4. Re-read the question.
5. Refer to your memory aid, as needed.
6. Solve the problem without looking at choices shown (A, B, C and D).
7. Look at all the choices.
8. Match your answer to the appropriate choice.

Do not leave a blank!  
Make a choice!

**My Strategies:**

**Answer and Solution:**

$$\tan x = \frac{28}{10}$$

$$x = \tan^{-1}(2.8) = 70.3^\circ$$

$$? = 180^\circ - 2(70.3^\circ)$$

$$= 39.4^\circ$$

(You may instead get the result  $39.3^\circ$  which would be correct.)

- A) Incorrect. Did  $\tan x = \frac{10}{28}$  and forgot to subtract from  $180^\circ$ .
- B) **Correct answer.**
- C) Incorrect. Forgot to subtract from  $180^\circ$ .
- D) Incorrect. Did  $\tan x = \frac{10}{28}$  and subtracted from  $180^\circ$ .

**The answer is B.**

**Suggested Strategies:**

- Identify the triangles that you see – two right triangles and an isosceles triangle.
- Determine if the two right triangles are the same. (they are by SAS – the two legs and the right angles)
- Since you know the lengths of two sides of the right triangle, you can use a trig ratio to find the angle(s)
- You know that the sum of the angles along the side of the rectangle measuring 20 must add up to 180.
- Find  $x$  and subtract that twice from 180 and you'll get the measure of the missing angle.

**Additional Resources:**

Visions Volume 1 p. 95 (Mathematical Knowledge Summary)

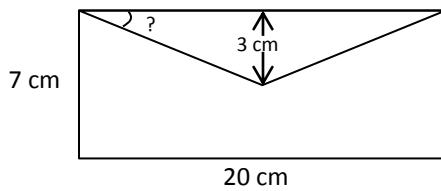
Explore Learning Gizmos, <http://www.explorellearning.com/> look up: Proving Sine, Cosine and Tangent

Khan Academy video: [http://www.khanacademy.org/math/geometry/right\\_triangles\\_topic/ccgeometry-trig/v/basic-trigonometry](http://www.khanacademy.org/math/geometry/right_triangles_topic/ccgeometry-trig/v/basic-trigonometry) Sine, Cosine and Tangent Trigonometric Functions

**Question:**

A stationary supplier needs to calculate the angle formed by the flap as shown in the diagram. The envelope is 20 cm long and 7 cm wide. The flap folds over a maximum of 3 cm at its center.

What is the measure of this angle?



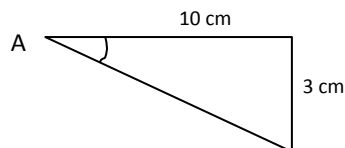
- A)  $16.7^\circ$
- B)  $17.5^\circ$
- C)  $19.3^\circ$
- D)  $20.5^\circ$

**General Strategies:**

1. Read the question.
2. Highlight key words.
3. Identify the math topic.
4. Re-read the question.
5. Refer to your memory aid, as needed.
6. Solve the problem without looking at choices shown (A, B, C and D).
7. Look at all the choices.
8. Match your answer to the appropriate choice.

Do not leave a blank!  
Make a choice!

**My Strategies:**

**Answer and Solution:**

$$\tan A = \frac{3}{10}$$

$$\angle A = \tan^{-1}\left(\frac{3}{10}\right)$$

$$\angle A = 16.7^\circ$$

- A) **Correct Answer.**
- B) Incorrect. Used  $\sin \frac{3}{10}$  instead of  $\tan \frac{3}{10}$ .
- C) Incorrect. Used  $\tan \frac{7}{20}$  instead of  $\tan \frac{3}{10}$ .
- D) Incorrect. Used  $\sin \frac{7}{20}$  instead of  $\tan \frac{3}{10}$ .

The answer is A.

**Suggested Strategies:**

- Look at the illustration and find the right triangle(s) – you should see the two (implied) right triangles in the obtuse triangle of the flap.
- Since the 3 cm goes down the center, the top can be divided into 10 cm and 10 cm and you have two right triangles.
- (Notice that the 7 cm measure is extra information that you don't need.)
- Now you're ready to choose a trig ratio to solve for the missing angle. Tangent is what works in this case.
- Turn your paper around if the orientation of the triangle is confusing.

**Additional Resources:**

Visions Volume 2 p. 95 (Mathematical Knowledge Summary)

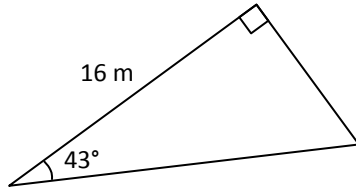
Explore Learning Gizmos, <http://www.explorelearning.com/> look up: Proving Sine, Cosine and Tangent

Khan Academy video: [http://www.khanacademy.org/math/geometry/right\\_triangles\\_topic/ccgeometry-trig/v/basic-trigonometry](http://www.khanacademy.org/math/geometry/right_triangles_topic/ccgeometry-trig/v/basic-trigonometry) Sine, Cosine and Tangent Trigonometric Functions

**Question:**

Sylvain wants to paint the surface of his triangular deck.

What is the area to be painted?



- A)  $87.3 \text{ m}^2$
- B)  $93.6 \text{ m}^2$
- C)  $119.4 \text{ m}^2$
- D)  $137.3 \text{ m}^2$

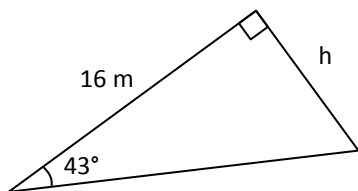
**General Strategies:**

1. Read the question.
2. Highlight key words.
3. Identify the math topic.
4. Re-read the question.
5. Refer to your memory aid, as needed.
6. Solve the problem without looking at choices shown (A, B, C and D).
7. Look at all the choices.
8. Match your answer to the appropriate choice.

Do not leave a blank!  
Make a choice!

**My Strategies:**



**Answer and Solution:**C)  $119.4 \text{ m}^2$ 

$$\tan 43^\circ = \frac{h}{16}$$

$$0.9325 = \frac{h}{16}$$

$$h = 16 \times 0.9325$$

$$h = 14.92 \text{ m}$$

$$A = \frac{16 \times 14.92}{2} = 119.4 \text{ m}^2$$

- A) Incorrect.  $\sin 43^\circ$  was used instead of  $\tan 43^\circ$ .
- B) Incorrect.  $\cos 43^\circ$  was used instead of  $\tan 43^\circ$ .
- C) Correct answer.**
- D) Incorrect.  $\tan 47^\circ$  was used instead of  $\tan 43^\circ$ .

The answer is C.

**Suggested Strategies:**

- Finding the area of a triangle means we need either the measure of all three sides (and use Hero's formula) or we need a base and its height.
- Here we have one side and an angle...in a right triangle so trig ratios can be used to find the measures of missing sides.
- If we consider the 16 m side as the base, the height is opposite the  $43^\circ$  angle.
- Tangent is the trig ratio to use in this case.

**Additional Resources:**

Visions Volume 2 p. 95 (Mathematical Knowledge Summary)

Explore Learning Gizmos, <http://www.explorelarning.com/> look up: Proving Sine, Cosine and Tangent

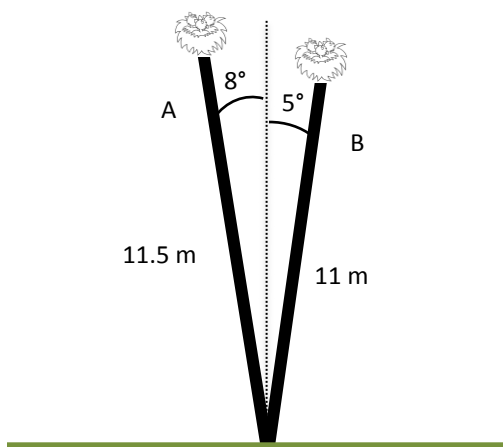
Khan Academy video: [http://www.khanacademy.org/math/geometry/right\\_triangles\\_topic/ccgeometry-trig/v/basic-trigonometry](http://www.khanacademy.org/math/geometry/right_triangles_topic/ccgeometry-trig/v/basic-trigonometry) Sine, Cosine and Tangent Trigonometric Functions

**Question:**

Bird nests are sitting at the top of two poles. Pole A is 11.5 m long and is leaning at an  $8^\circ$  angle from the vertical; Pole B is 11 m long and is leaning at a  $5^\circ$  angle from the vertical.

What is the difference in height between the two bird nests?

Round your answer to the nearest hundredth.



The difference in height between the two bird nests is \_\_\_\_\_ m.

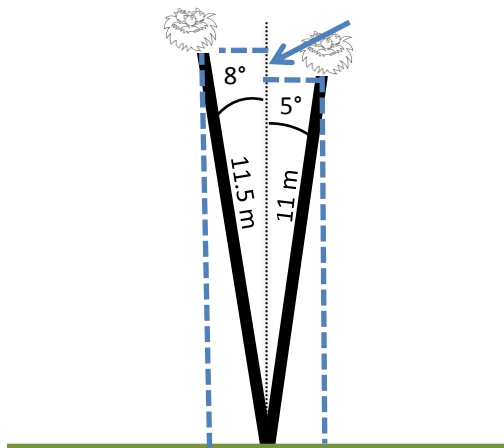
**General Strategies:**

1. Read the question.
  2. Highlight key words.
  3. Identify the math topic.
  4. Re-read the question.
  5. Make a prediction about the answer- what will it look like? (an equation, a number, etc.).
  6. Refer to your memory aid, as needed.
  7. Solve.
  8. Ask yourself whether your answer makes sense.
  9. Write your answer.
- Do not leave a blank!

**My Strategies:**

**Answer and Solution:**

Difference in height between the bird nests = 0.43 m



Pole A	Pole B
$\cos 8^\circ = \frac{\text{Height}_A}{11.5}$	$\cos 5^\circ = \frac{\text{Height}_B}{11}$
$0.990 = \frac{\text{Height}_A}{11.5}$	$0.996(11) = \text{Height}_B$
$\text{Height}_A = 11.39 \text{ m}$	$\text{Height}_B = 10.96 \text{ m}$

Difference is  $11.39 - 10.96 = \underline{0.43 \text{ m}}$

The difference in height between the two bird nests is 0.43 m.

**Suggested Strategies:**

- Before you can find the difference in the heights of the bird nests, you need to find the height from the ground of each one.
- Height is a vertical distance so the height of each nest will be less than the length of the poles.
- You will have to identify right triangles and their elements before you can use trig ratios to solve for the missing measure.

**Additional Resources:**

Visions Volume 2 p. 95 (Mathematical Knowledge Summary)

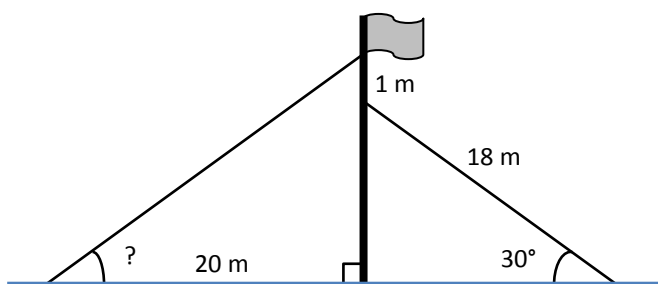
Explore Learning Gizmos, <http://www.explorelearning.com/> look up: Proving Sine, Cosine and Tangent

Khan Academy video: [http://www.khanacademy.org/math/geometry/right\\_triangles\\_topic/ccgeometry-trig/v/basic-trigonometry](http://www.khanacademy.org/math/geometry/right_triangles_topic/ccgeometry-trig/v/basic-trigonometry) Sine, Cosine and Tangent Trigonometric Functions

**Question:**

A flagpole is anchored using two guy wires. The guy wire on the right is 18 m long and has an angle of inclination with the ground of  $30^\circ$ . It is attached 1 m below the point where the left guy wire is attached to the pole. The left guy wire is located 20 m from the base of the flagpole.

What is the angle of inclination of the left guy wire?

**General Strategies:**

1. Read the question.
2. Highlight key words.
3. Identify the math topic.
4. Re-read the question.
5. Make a prediction about the answer- what will it look like? (an equation, a number, etc.).
6. Refer to your memory aid, as needed.
7. Solve.
8. Ask yourself whether your answer makes sense.
9. Write your answer. Do not leave a blank!

**My Strategies:**

The angle of inclination of the left guy wire is \_\_\_\_\_.

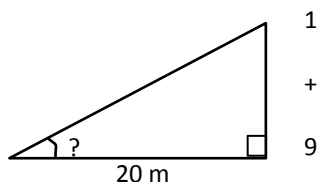
**Answer and Solution:**

$$\sin 30^\circ = \frac{x}{18}$$

$$0.5 = \frac{x}{18}$$

$$x = 0.5 \times 18 = 9 \text{ m}$$

or the length of the side opposite a  $30^\circ$  angle in a  $30^\circ$ - $60^\circ$ - $90^\circ$  triangle is  $\frac{1}{2}$  the length of the hypotenuse



$$y = \tan^{-1}\left(\frac{9}{20}\right)$$

$$y = \tan^{-1}(0.45)$$

$$y = 26.6^\circ$$

The angle of inclination of the left guy wire is  $26.6^\circ$ .

**Suggested Strategies:**

- Determine what you need to know about the left triangle in order to find the missing angle measure.
  - Either the hypotenuse (the length of the guy wire),
  - the other angle or
  - the height up the flagpole of the other guy wire.
- We only have enough information to find the last option.

**Additional Resources:**

Visions Volume 2 p. 95 (Mathematical Knowledge Summary)

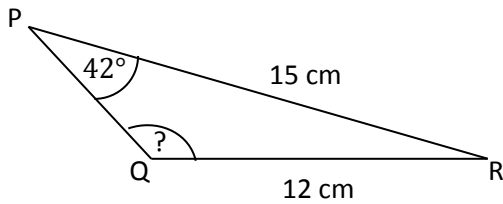
Explore Learning Gizmos, <http://www.explorelearning.com/> look up: Proving Sine, Cosine and Tangent

Khan Academy video: [http://www.khanacademy.org/math/geometry/right\\_triangles\\_topic/ccgeometry-trig/v/basic-trigonometry](http://www.khanacademy.org/math/geometry/right_triangles_topic/ccgeometry-trig/v/basic-trigonometry) Sine, Cosine and Tangent Trigonometric Functions

**Question:**

In triangle PQR, we have  $m\angle P = 42^\circ$ ;  $m\overline{PR} = 15\text{ cm}$ ;  $m\overline{QR} = 12\text{ cm}$

In addition,  $\angle PQR$  is an obtuse angle.



To the nearest degree, what is the measure of  $\angle PQR$ ?

**General Strategies:**

1. Read the question.
2. Highlight key words.
3. Identify the math topic.
4. Re-read the question.
5. Make a prediction about the answer- what will it look like? (an equation, a number, etc.).
6. Refer to your memory aid, as needed.
7. Solve.
8. Ask yourself whether your answer makes sense.
9. Write your answer. Do not leave a blank!

**My Strategies:**

The measure of  $\angle PQR$  is \_\_\_\_\_.

**Answer and Solution:**

Use Sine Law to find  $m\angle PQ$

$$\frac{p}{\sin P} = \frac{q}{\sin Q} = \frac{r}{\sin R}$$

$$\frac{12 \text{ cm}}{\sin 42^\circ} = \frac{15 \text{ cm}}{\sin Q}$$

$$\sin Q = \frac{15 \text{ cm} \cdot \sin 42^\circ}{12 \text{ cm}}$$

$$\sin Q = 0.8364132579$$

Find the inverse of Sine, use

$$\sin^{-1}(0.8364132579) = 56.733^\circ$$

Careful, this is not an obtuse angle!!

Extra step – since we are looking for an obtuse angle...

$$\sin Q = \sin(180^\circ - Q)$$

$$\sin(56.7633^\circ) = \sin(180^\circ - 56.7633^\circ)$$

$$180^\circ - 56.7633^\circ = 123.2367^\circ$$

**The measure of  $\angle PQR$  is  $123^\circ$ .**

**Suggested Strategies:**

- Since this is not a right triangle, you cannot use SOH CAH TOA
- Don't forget that when you use Sine Law to determine the measure of an obtuse angle, you must subtract your answer from 180.

**Additional Resources:**

Visions Volume 2 p. 95 (Mathematical Knowledge Summary)

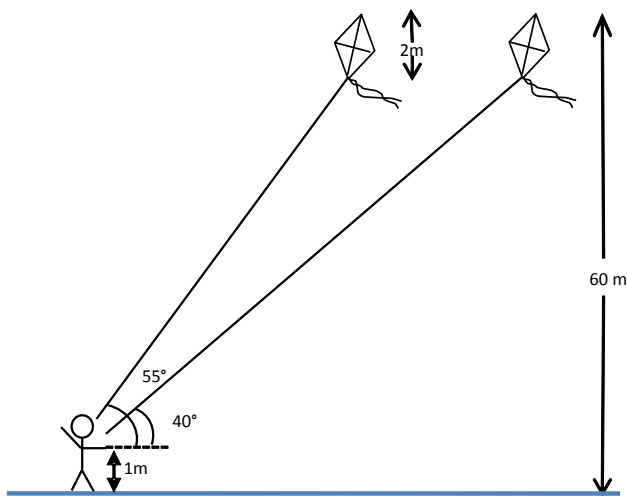
Explore Learning Gizmos, <http://www.explorellearning.com/> look up: Proving Sine, Cosine and Tangent

Khan Academy video: [http://www.khanacademy.org/math/geometry/right\\_triangles\\_topic/ccgeometry-trig/v/basic-trigonometry](http://www.khanacademy.org/math/geometry/right_triangles_topic/ccgeometry-trig/v/basic-trigonometry) Sine, Cosine and Tangent Trigonometric Functions

**Question:**

Sally is flying a kite. The tip of the kite is 60 m above the ground and the kite itself is 2 m in length. She is holding the string 1 m above the ground. The angle of inclination of the string started out at  $55^\circ$  but then the wind shifted and the angle of the string decreased to  $40^\circ$ . In order to maintain the height of the kite, Sally had to let more string out from the spool.

How much string did Sally need to let out to maintain the height of the kite?



Sally had to let out an additional \_\_\_\_\_ of string to maintain the height of the kite.

**General Strategies:**

1. Read the problem.
2. Highlight key words.
3. Identify the math topics.
4. Re-read the problem.
5. Define your steps (your game plan) – this is criteria 3.
6. Refer to your memory aid, as needed.
7. Solve.
8. If you get stuck on a calculation, make an educated guess and keep going.
9. Ask yourself whether your answer makes sense.
10. Write your answer statement.

Show any or all your work! Do not leave a blank page!

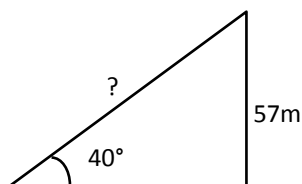
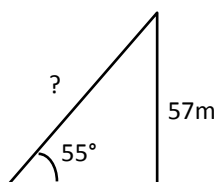
**My Strategies:**



**Answer and Solution:**

Height of kite where the string is attached is  
 $60\text{ m} - 2\text{ m} = 58\text{ m}$

Sally is holding the string 1 m up so height for calculation is  $58\text{ m} - 1\text{ m} = 57\text{ m}$



Length of string originally:

$$\begin{aligned} \sin 55^\circ &= \frac{57\text{ m}}{l_1} \\ 0.8191 &= \frac{57\text{ m}}{l_1} \\ l_1 &= \frac{57\text{ m}}{0.8191} = 69.58\text{ m} \end{aligned}$$

Length of string after the wind shift:

$$\begin{aligned} \sin 40^\circ &= \frac{57\text{ m}}{l_2} \\ 0.6428 &= \frac{57\text{ m}}{l_2} \\ l_2 &= \frac{57}{0.6428} = 88.68\text{ m} \end{aligned}$$

$$88.68\text{ m} - 69.58\text{ m} = 19.1\text{ m}$$

**Sally had to let out an additional 19.1 m of string to maintain the height of the kite.**

**Suggested Strategies:**

- The tricky part of this question is trying to imagine what exactly is happening.
- From the diagram, you can probably see that triangles are involved; sketch them separately in a way that will make it easier to see how to set up the trig ratios
- Recognize that the string is the hypotenuse of these triangles and get the feel that the hypotenuse will be longer if the angle is smaller.
- Now figure out the two lengths and find the difference.

**Additional Resources:**

Visions Volume 2 p. 95 (Mathematical Knowledge Summary)

Explore Learning Gizmos, <http://www.explorelarning.com/> look up: Proving Sine, Cosine and Tangent

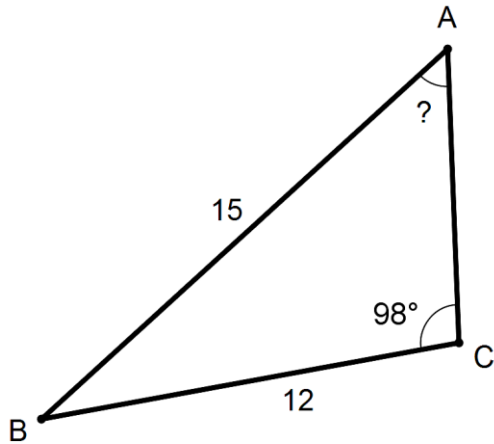
Khan Academy video: [http://www.khanacademy.org/math/geometry/right\\_triangles/topic/ccgeometry-trig/v/basic-trigonometry](http://www.khanacademy.org/math/geometry/right_triangles/topic/ccgeometry-trig/v/basic-trigonometry) Sine, Cosine and Tangent Trigonometric Functions



## **5.3 Calculating the Area of any Triangle**

**Question:**

Consider the following diagram of triangle ABC.



What is the measure of angle CAB?

- A) 53.1°
- B) 52.4°
- C) 38.7°
- D) 36.8°

**General Strategies:**

1. Read the question.
2. Highlight key words.
3. Identify the math topic.
4. Re-read the question.
5. Refer to your memory aid, as needed.
6. Solve the problem without looking at choices shown (A, B, C and D).
7. Look at all the choices.
8. Match your answer to the appropriate choice.

Do not leave a blank!  
Make a choice!

**My Strategies:**

**Answer and Solution:**

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$

$$\frac{12}{\sin A} = \frac{15}{\sin 98}$$

$$12(\sin 98) = 15 \sin A$$

$$\sin A = 0.79$$

$$A = 52.39^\circ$$

- A)  $53.1^\circ$ ; Did not use sine law, used  $\sin CAB = \frac{a}{c}$  instead.
- B)  $52.4^\circ$ ; Correct Answer.**
- C)  $38.7^\circ$ ; Did not use sine law, used  $\tan CAB = \frac{a}{c}$  instead.
- D)  $36.8^\circ$ ; Did not use sine law, used  $\cos CAB = \frac{a}{c}$  instead.

**Specific Strategies:**

- Notice that this is not a right angle ( $90^\circ$ ) triangle.
- You cannot apply SOH, CAH, TOA.
- You should use Sine law.
- Color code, highlight or match the angles with their corresponding sides.
- Apply Sine law formula.

The answer is B.

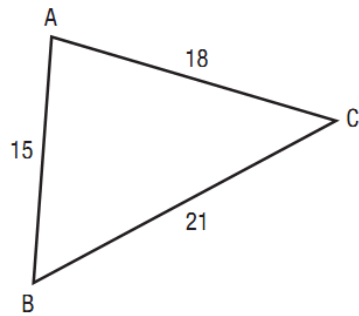
**Additional Resources:**

Visions Volume 2, Section 5.3, p. 108

Khan Academy video: <https://www.khanacademy.org/math/trigonometry/less-basic-trigonometry/law-sines-cosines/v/law-of-sines> Law of Sines

**Question:**

Consider the following diagram of triangle ABC. All measurements are in metres.



What is the area of triangle ABC?

- A)  $7.35 \text{ m}^2$
- B)  $25.5 \text{ m}^2$
- C)  $132.3 \text{ m}^2$
- D)  $187.1 \text{ m}^2$

**General Strategies:**

1. Read the question.
2. Highlight key words.
3. Identify the math topic.
4. Re-read the question.
5. Refer to your memory aid, as needed.
6. Solve the problem without looking at choices shown (A, B, C and D).
7. Look at all the choices.
8. Match your answer to the appropriate choice.

Do not leave a blank!  
Make a choice!

**My Strategies:**

**Answer and Solution:**

$$p = \frac{(a + b + c)}{2}$$

$$p = \frac{(21 + 18 + 15)}{2} = 27$$

$$A = \sqrt{p(p - a)(p - b)(p - c)}$$

$$A = \sqrt{27(27 - 21)(27 - 18)(27 - 15)}$$

$$A = \sqrt{27(6)(9)(12)}$$

$$A = \sqrt{17\,496}$$

$$A = 132.3 \text{ m}^2$$

$$\text{Area} = 132.3 \text{ m}^2$$

- A)  $7.35 \text{ m}^2$ ; added the values under the radical
- B)  $25.5 \text{ m}^2$ ; Forgot to multiply everything by  $p$  under the radical.
- C)  $132.3 \text{ m}^2$ ; Correct Answer**
- D)  $187.1 \text{ m}^2$ ; Multiplied everything by the perimeter instead of the half perimeter

The answer is C.

**Specific Strategies:**

Identify the values of  $a$ ,  $b$  and  $c$  according to the diagram provided.

Calculate the value of the half perimeter.

Substitute the values for  $a$ ,  $b$ ,  $c$ , and  $p$  in Hero's formula.

**Additional Resources:**

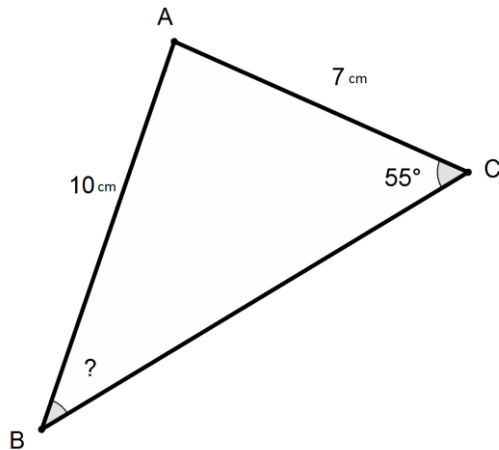
Visions Volume 2, Section 5.3, p. 108

Khan Academy video: <https://www.khanacademy.org/math/geometry/basic-geometry/heron-formulatutorial/v/heron-s-formula> Heron's Formula

**Question:**

Consider triangle ABC shown below.

To the nearest degree, what is the measure of angle  $AB$ ?

**General Strategies:**

1. Read the question.
  2. Highlight key words.
  3. Identify the math topic.
  4. Re-read the question.
  5. Make a prediction about the answer- what will it look like? (an equation, a number, etc.).
  6. Refer to your memory aid, as needed.
  7. Solve.
  8. Ask yourself whether your answer makes sense.
  9. Write your answer.
- Do not leave a blank!

**My Strategies:**

The measure of angle  $ABC$  is \_\_\_\_\_.



**Answer and Solution:**

Knowing that angle C is  $55^\circ$ , that side C has a length of 10 and side b a length of 7, we can calculate the size of angle B using sine law.

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

To solve the problem, we need three of the four values in any given equality. Since we know b, c and  $\sin C$ , then:

$$\frac{b}{\sin B} = \frac{c}{\sin C}$$

Rearranging the equation we get:

$$\sin B = \frac{b \sin C}{c}$$

Substituting the corresponding values, we get:

$$\sin B = \frac{7 \sin 55^\circ}{10}$$

$$\sin B = 0.573406431$$

Finding the inverse of  $\sin$ ,  $\sin^{-1}$ , we can get the measure of the angle:

$$B = \sin^{-1} 0.573406431$$

$$B = 35^\circ$$

**Answer: Angle B =  $35^\circ$**

**Specific Strategies:**

- Notice that this is not a right angle ( $90^\circ$ ) triangle
- You cannot apply SOH, CAH, TOA
- You should use Sine law
- Color code, highlight or match the angles with their corresponding sides
- Apply Sine law formula

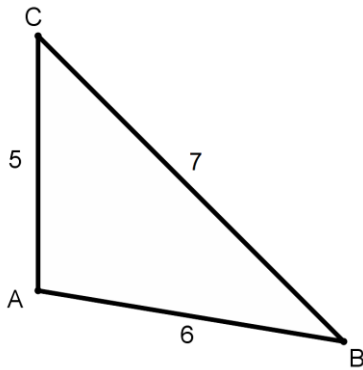
**Additional Resources:**

Visions Volume 2, Section 5.3, p. 103 (Activity 1: Sine Law)

Khan Academy video: <https://www.khanacademy.org/math/trigonometry/less-basic-trigonometry/law-sines-cosines/v/law-of-sines> Law of Sines

**Question:**

What is the area of triangle ABC shown below?  
All measurements are in cm.

**General Strategies:**

1. Read the question.
  2. Highlight key words.
  3. Identify the math topic.
  4. Re-read the question.
  5. Make a prediction about the answer- what will it look like? (an equation, a number, etc.).
  6. Refer to your memory aid, as needed.
  7. Solve.
  8. Ask yourself whether your answer makes sense.
  9. Write your answer.
- Do not leave a blank!

**My Strategies:**

The area of triangle ABC is \_\_\_\_\_ .

**Answer and Solution:**

Before starting with Hero's formula, we must first determine the half-perimeter  $p$  of the triangle.

$$\text{Since } p = \frac{(a + b + c)}{2}$$

where  $a = 7 \text{ cm}$ ,  $b = 5 \text{ cm}$  and  $c = 6 \text{ cm}$

$$\text{we know that } p = \frac{(7 \text{ cm} + 5 \text{ cm} + 6 \text{ cm})}{2} = 9 \text{ cm}$$

Now using the formula we get:

$$A = \sqrt{p(p - a)(p - b)(p - c)}$$

$$A = \sqrt{9(9 - 7)(9 - 5)(9 - 6)}$$

$$A = \sqrt{9(2)(4)(3)}$$

$$A = \sqrt{216}$$

$$A = 14.7 \text{ cm}^2$$

$$\text{Area} = 14.7 \text{ cm}^2$$

The answer is  $14.7 \text{ cm}^2$ .

**Specific Strategies:**

Identify the values of  $a$ ,  $b$  and  $c$  according to the diagram provided.

Calculate the value of the half perimeter.

Substitute the values for  $a$ ,  $b$ ,  $c$ , and  $p$  in Hero's formula.

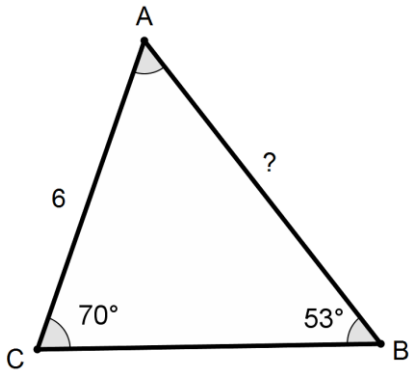
**Additional Resources:**

Visions Volume 2 Activity 2 p. 105 and Mathematical Knowledge p. 108

Khan Academy video: [http://www.khanacademy.org/math/geometry/right\\_triangles\\_topic/cc-geometry-trig/v/basic-trigonometry](http://www.khanacademy.org/math/geometry/right_triangles_topic/cc-geometry-trig/v/basic-trigonometry) Sine, Cosine and Tangent Trigonometric Functions

**Question:**

Consider triangle ABC shown below. What is the length of segment AB? Round your answer to the nearest tenth.

**General Strategies:**

1. Read the question.
2. Highlight key words.
3. Identify the math topic.
4. Re-read the question.
5. Make a prediction about the answer- what will it look like? (an equation, a number, etc.).
6. Refer to your memory aid, as needed.
7. Solve.
8. Ask yourself whether your answer makes sense.
9. Write your answer.  
Do not leave a blank!

**My Strategies:**

The length of segment AB is \_\_\_\_\_.

**Answer and Solution:**

Knowing that angle B is  $53^\circ$ , that side b has a length of 6 and angle C is  $70^\circ$ , we can calculate the length of side c using sine law.

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

To solve the problem, we need three of the four values in any given equality. Since we know b, sin B and sin C, then:

$$\frac{b}{\sin B} = \frac{c}{\sin C}$$

Rearranging the equation we get:

$$c = \frac{b \sin C}{\sin B}$$

Substituting the corresponding values, we get:

$$c = \frac{6 \sin 70^\circ}{\sin 53^\circ}$$

$$c = 7.06$$

Since side c corresponds to AB, the length of segment AB is 7.06.

**Answer: the length of segment AB is 7.06.**

**Specific Strategies:**

- Notice that this is not a right angle ( $90^\circ$ ) triangle
- You cannot apply SOH, CAH, TOA
- You should use Sine law
- Color code, highlight or match the angles with their corresponding sides
- Apply Sine law formula

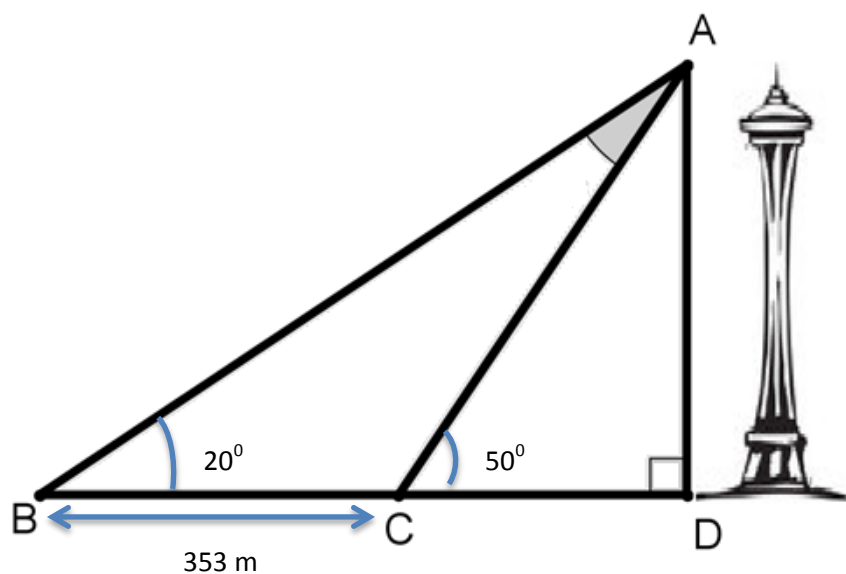
**Additional Resources:**

Visions Volume 2, Section 5.3, p. 103 ( Activity 1: Sine Law)

Khan Academy video: [http://www.khanacademy.org/math/geometry/right\\_triangles\\_topic/cc-geometry-trig/v/basic-trigonometry](http://www.khanacademy.org/math/geometry/right_triangles_topic/cc-geometry-trig/v/basic-trigonometry) Sine, cosine and tangent trigonometric functions

**Question:**

The Space Needle is a tall structure in Seattle, Washington. Phil, a math student, attempts to estimate the height of the Space Needle by using a clinometer, a device that measures the angle of inclination.



First, Phil stands at point C and reads a  $50^\circ$  angle on the clinometer. Then, Phil moves 353 m to Point B and reads an angle of  $20^\circ$  on the clinometer. Phil estimates the Space Needle is between 182 m and 188 m in height.

Based on the information given, is Phil's estimation correct? Explain.

- Yes, his estimation is correct.
- No, his estimation is not correct.

**General Strategies:**

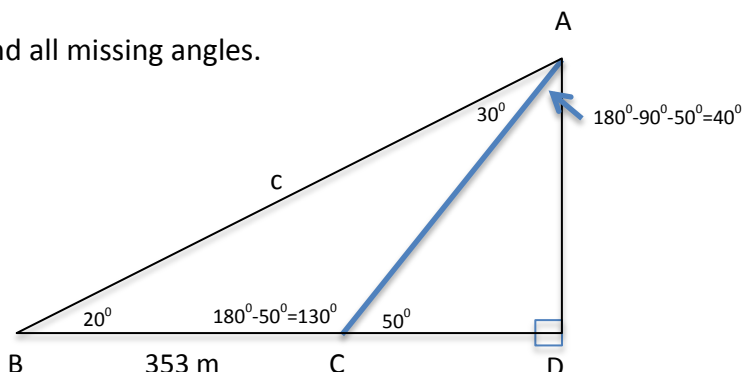
1. Read the problem.
2. Highlight key words.
3. Identify the math topics.
4. Re-read the problem.
5. Define your steps (your game plan) – this is criteria 3.
6. Refer to your memory aid, as needed.
7. Solve.
8. If you get stuck on a calculation, pick a number and keep going.
9. Ask yourself whether your answer makes sense.
10. Write your answer statement.

Show any or all your work! Do not leave a blank page!

**My Strategies:**

**Answer and Solution:**

Find all missing angles.



Apply Sine law to find  $c$ .

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$

$$\frac{353}{\sin 30} = \frac{c}{\sin 130}$$

$$c = 541 \text{ m}$$

Apply trig to find  $\overline{AD}$  (height of Space Needle).

$$\sin 20 = \frac{\overline{AD}}{541}$$

$$\overline{AD} = 185 \text{ m}$$

- Yes, his estimation is correct.  
 No, his estimation is not correct.

Reason: The clinometer measures yield a height of 185 m for the Space Needle. This falls within Phil's estimation of 182 m to 188 m.

**Specific Strategies:**

1. Find all the missing angles you can
2. Notice triangle ABD is a right angle triangle composed of two other triangles
3. Triangle ABC is not a right angle triangle. So, apply Sine law
4. Triangle ACD is a right angle triangle. So, apply SOH, CAH TOA

**Additional Resources:**

Visions Volume 2, Section 5.3, p. 103 (Activity 1: Sine Law)

Khan Academy video: <https://www.khanacademy.org/math/trigonometry/less-basic-trigonometry/law-sines-cosines/v/law-of-sines> Law of Sines





## **6.2 Subjective Probability and Odds**

**Question:**

Which of the following is an example of subjective probability?

- A) You are rolling a die. The probability of rolling a 4 is  $\frac{1}{6}$ .
- B) A camera records the cars passing through an intersection. The probability that the next car will be red is 0.05.
- C) You are waiting for a bus. The probability of it being late is 10%.
- D) You are sitting with your doctor hearing the results of various diagnostic tests. The doctor gives an approximation of your life expectancy.

**General Strategies:**

1. Read the question.
  2. Highlight key words.
  3. Identify the math topic.
  4. Re-read the question.
  5. Make a prediction about the answer- what will it look like? (an equation, a number, etc.).
  6. Refer to your memory aid, as needed.
  7. Solve.
  8. Ask yourself whether your answer makes sense.
  9. Write your answer.
- Do not leave a blank!

**My Strategies:**

**Answer and Solution:**

- A) is theoretical probability.
- B) is experimental/empirical probability.
- C) is experimental/empirical probability.
- D) is subjective probability.**

The answer is D.

**Suggested Strategies:**

- Look up the definitions.
- Determine which answer is an example of subjective probability.

**Additional Resources:**

Visions Volume 2, p. 152

**Question:**

What are the odds for rolling a 4 on a fair die?

- A) 5:1
- B) 1:5
- C) 1:6
- D) 5:6

**General Strategies:**

1. Read the question.
  2. Highlight key words.
  3. Identify the math topic.
  4. Re-read the question.
  5. Make a prediction about the answer- what will it look like? (an equation, a number, etc.).
  6. Refer to your memory aid, as needed.
  7. Solve.
  8. Ask yourself whether your answer makes sense.
  9. Write your answer.
- Do not leave a blank!

**My Strategies:**

**Answer and Solution:**

“Odds for” is the ratio of favorable outcome to unfavorable outcomes (favorable : unfavorable).

Favorable means the outcome you are looking for and unfavorable refers to all other outcomes.

We have one 4 on a die and five other numbers which are not 4, therefore the odds for rolling a 4 are 1:5.

- A) 5:1 odds against rolling a 4
- B) **1:5 odds for rolling a 4**
- C) 1:6 probability of rolling a 4
- D) 5:6 probability of not rolling a 4

The answer is B.

**Suggested Strategies:**

- Determine meaning of “odds for.”
- Determine the number of fours you can obtain on a fair die.

**Additional Resources:**

Visions Volume 2, p. 153

**Question:**

A boxer has a 30% chance of winning the championship.

What are his odds against winning?

The boxer's odds against winning are \_\_\_\_\_.

**General Strategies:**

1. Read the question.
  2. Highlight key words.
  3. Identify the math topic.
  4. Re-read the question.
  5. Make a prediction about the answer- what will it look like? (an equation, a number, etc.).
  6. Refer to your memory aid, as needed.
  7. Solve.
  8. Ask yourself whether your answer makes sense.
  9. Write your answer.
- Do not leave a blank!

**My Strategies:**

**Answer and Solution:**

$$30\% = \frac{30}{100} = \frac{3}{10} = \frac{\text{favorable}}{\text{total}}$$

$$\begin{aligned}\text{Unfavorable} &= \text{total} - \text{favorable} \\ &= 10 - 3 \\ &= 7\end{aligned}$$

“Odds against” is the ratio

$$\text{unfavorable} : \text{favorable} \rightarrow 7:3$$

**Suggested Strategies:**

- Determine the fraction associated with chances of winning.
- Use those numbers to determine the values for favorable and unfavorable.
- Determine the ratio for the odds against.

**Answer:** The boxer’s odds against winning are 7:3.

**Additional Resources:**

Visions Volume 2, p. 153

**Question:**

Sam bets \$12 on a horse race. The odds in favor of his horse winning are 4:5.

How much money would Sam collect if his horse wins the race?

Sam would collect \$\_\_\_\_\_ if his horse wins.

**General Strategies:**

1. Read the question.
  2. Highlight key words.
  3. Identify the math topic.
  4. Re-read the question.
  5. Make a prediction about the answer- what will it look like? (an equation, a number, etc.).
  6. Refer to your memory aid, as needed.
  7. Solve.
  8. Ask yourself whether your answer makes sense.
  9. Write your answer.
- Do not leave a blank!

**My Strategies:**



**Answer and Solution:**

$$\frac{4}{5} = \frac{12}{x}$$

$$x = \$15$$

$$\$15 \text{ win} + \$12 \text{ bet} = \$27 \text{ remittance}$$

Or

$$\frac{\text{chances of win}}{5} = \frac{\text{amount bet}}{\text{amount won}}$$

$$\frac{4}{5} = \frac{12}{x}$$

$$x = \$15$$

$$\$15 \text{ win} + \$12 \text{ bet} = \$27 \text{ remittance}$$

**Answer:** Sam will receive \$27 if his horse wins.

**Suggested Strategies:**

- Determine the proportion for calculating bets.
- Solve for the missing term.
- Don't forget the initial amount he bets.
- Don't forget the amount he initially bets.

**Additional Resources:**

Visions Volume 2, p. 153

**Question:**

A bag contains 20 marbles that are either blue or yellow.

If a marble is drawn at random from the bag, the odds that the marble will be blue are 1 to 4.

How many yellow marbles are in the bag?

There are \_\_\_\_\_ yellow marbles in the bag.

**General Strategies:**

1. Read the question.
  2. Highlight key words.
  3. Identify the math topic.
  4. Re-read the question.
  5. Make a prediction about the answer- what will it look like? (an equation, a number, etc.).
  6. Refer to your memory aid, as needed.
  7. Solve.
  8. Ask yourself whether your answer makes sense.
  9. Write your answer.
- Do not leave a blank!

**My Strategies:**

**Answer and Solution:**

Odds for blue is the ratio  
favoring blue : against blue  
1 : 4

The probability of drawing a blue marble is :

$$\frac{\textit{favoring blue}}{\textit{total}} = \frac{1}{1 + 4} = \frac{1}{5}$$

$$\frac{1}{5} = \frac{x}{20}$$

$x = 4$  blue marbles

$20 - 4 = 16$  yellow marbles

**Answer:** There are 16 yellow marbles in the bag.

**Suggested Strategies:**

- Relate the given “odds” with a probability so that you can determine the number of blue marbles, and from there, the number of yellow marbles.

**Additional Resources:**

Visions Volume 2, p. 153



## **6.3 Mathematical Expectation**

**Question:**

Ralph bets \$5 to play a game that involves rolling a die. If he rolls a 5 or higher, he receives \$10 plus his bet; otherwise, he loses his bet.

Which statement is true?

- A) The game is to the player's advantage.
- B) The game is fair.
- C) The game is to the player's disadvantage.
- D) The game's fairness cannot be determined.

**General Strategies:**

1. Read the question.
2. Highlight key words.
3. Identify the math topic.
4. Re-read the question.
5. Refer to your memory aid, as needed.
6. Look carefully at each choice shown (A, then B, then C and then D).
7. Eliminate options you know to be incorrect.
8. Solve/check each possible choice.
9. Select the choice that makes the most sense.

Do not leave a blank!

Make a choice!

**My Strategies:**

**Answer and Solution:**

Mathematical expectation =  $(P_1 \times O_1) + (P_2 \times O_2)$

$$\text{Mathematical expectation} = \frac{2}{6}(10) + \frac{4}{6}(-5) = 0$$

Because the mathematical expectation is 0, the game is fair.

Event of rolling a 5 or higher =  $\{5, 6\}$

$$P(\geq 5) = \frac{2}{6}$$

$$P(< 5) = \frac{4}{6}$$

$$\text{Outcome of } P(\geq 5) = (\$10 + \$5 - \$5) = \$10$$

$$\text{Outcome of } P(< 5) = -\$5$$

The answer is B.

**Suggested Strategies:**

- Find the probability of winning.
- Find the probability of losing.
- Find the profit you obtain from winning.
- Find the profit you obtain from losing.
- If the result is 0 then the game is fair.

**Additional Resources:**

Visions Volume 2, p. 164

**Question:**

A box contains ten \$5 bills and fifteen \$10 bills.

Players must bet \$8 in order to play the game.

Players keep the bill they draw from the box.

What is the expected gain of this game?

- A) -1
- B) 0
- C) 5
- D) 8

**General Strategies:**

1. Read the question.
2. Highlight key words.
3. Identify the math topic.
4. Re-read the question.
5. Refer to your memory aid, as needed.
6. Solve the problem without looking at choices shown (A, B, C and D).
7. Look at all the choices.
8. Match your answer to the appropriate choice.

Do not leave a blank!  
Make a choice!

**My Strategies:**



**Answer and Solution:**

Mathematical expectation =  $(P_1 \times O_1) + (P_2 \times O_2)$

$$P_1(\$10) = \left(\frac{10}{10 + 15}\right) = \frac{10}{25}$$

$$P_2(\$15) = \left(\frac{15}{10 + 15}\right) = \frac{15}{25}$$

$$\text{Mathematical expectation} = \frac{10}{25}(5 - 8) + \frac{15}{25}(10 - 8) = 0$$

$$\text{Mathematical expectation} = (-\$1.20) + (\$1.20) = 0$$

- A) -1 – if you mix up the probabilities
- B) **0 correct**
- C) 5 – if you mix up the probabilities and not consider the cost to play
- D) 8 – if you don't consider the cost to play

**The answer is B.**

**Suggested Strategies:**

- Find the probability of winning.
- Find the probability of losing.
- Find the profit you obtain from winning.
- Find the profit you obtain from losing.
- Calculate the expected gain.

**Additional Resources:**

Visions Volume 2, p. 164

**Question:**

A game involves throwing 2 coins.

If a player receives two heads, the player wins \$10.

If a player receives two tails, the player wins \$20.

If the player receives any other combination, the player loses \$5.

What is the mathematical expectation of this game?

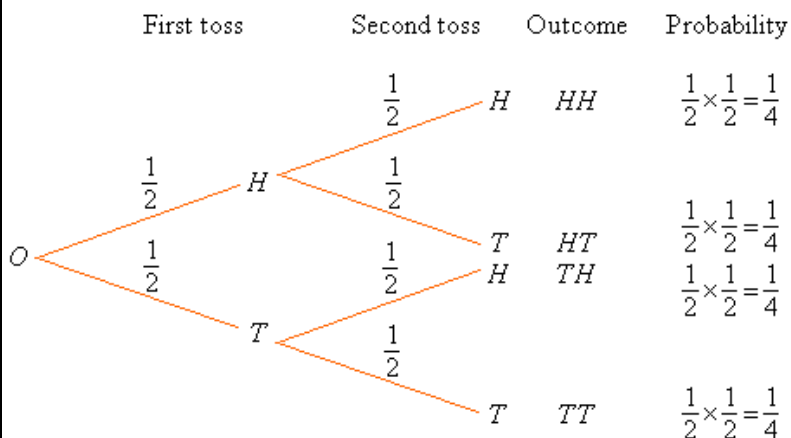
The mathematical expectation is \_\_\_\_\_.

**General Strategies:**

1. Read the question.
  2. Highlight key words.
  3. Identify the math topic.
  4. Re-read the question.
  5. Make a prediction about the answer- what will it look like? (an equation, a number, etc.).
  6. Refer to your memory aid, as needed.
  7. Solve.
  8. Ask yourself whether your answer makes sense.
  9. Write your answer.
- Do not leave a blank!

**My Strategies:**

**Answer and Solution:**



Mathematical expectation =  $P(H, H) \times O + (P, P) \times O + P(\text{lose}) \times O$

$$\frac{1}{4}(10) + \frac{1}{4}(20) + \frac{2}{4}(-5) = 5$$

The mathematical expectation is 5.

**Suggested Strategies:**

- Write the formula for expected value.
- Make a tree diagram for tossing two coins.
- Find the probability of winning.
- Find the probability of losing.
- Find the profit you obtain from winning.
- Find the profit you obtain from losing.
- Calculate the result.

**Additional Resources:**

Visions Volume 2, p. 164

**Question:**

It costs a smartphone manufacturer \$300 to make a phone.

The phones they make are either:

- Sold to cell phone service companies for \$550 each;
- Sold to retail stores for \$650 each;
- Sold to employees for a special price of \$500 each;
- Given away for promotional purposes (no cost); or
- Scrapped because they turn out to be defective.

Over the past year:

- 50% of the phones were sold to cell phone service companies;
- 45.5% were sold to retail stores;
- 1% were sold to employees;
- 0.5% were given away; and
- 3% were defective and not sold.

The company plans on manufacturing one million phones next year.

**What is the company's expected profit for next year?**

The company's expected profit for next year is \_\_\_\_\_.

**General Strategies:**

1. Read the question.
2. Highlight key words.
3. Identify the math topic.
4. Re-read the question.
5. Make a prediction about the answer- what will it look like? (an equation, a number, etc.).
6. Refer to your memory aid, as needed.
7. Solve.
8. Ask yourself whether your answer makes sense.
9. Write your answer.

Do not leave a blank!

**My Strategies:**

**Answer and Solution:**

outcome	cost	price	profit	probability	profit x probability
			$O_i$	$P_i$	$O_i \times P_i$
service company	\$300	\$550	\$250	50.0%	\$125
retail store	\$300	\$650	\$350	45.5%	\$159.25
employee	\$300	\$500	\$200	1.0%	\$2
give away	\$300	\$0	-\$300	0.5%	-\$1.5
defective	\$300	\$0	-\$300	3.0%	-\$9
total				100.0%	\$275.75

If there will be 1 000 000 phones produced, the expected profit would be  $\$275.75 \times 1\,000\,000 = \$275\,750\,000$ .

OR

$$\begin{aligned} \text{Expected Profit} &= (P_1 \times O_1) + (P_2 \times O_2) + (P_3 \times O_3) + (P_4 \times O_4) + (P_5 \times O_5) \\ &= (0.5 \times 259) + (0.445 \times 350) + (0.01 \times 200) + (0.005 \times -300) + (0.03 \times -300) \\ &= 275.75 / \text{phone} \times 1 \times 10^6 = 275\,750\,000 \end{aligned}$$

**The company's expected profit for next year is \$275 750 000.**

**Suggested Strategies:**

- These types of problems lend themselves very well to creating tables to keep track of all the subscripts in the formula.
- Identify outcomes – not just what they are but the result (in this case, where the phones get sold determine the profit for those phones.)
- Identify probabilities of each outcome
- Remember that the formula would give you the expected value for one phone – don't forget to multiply by the number of phones.

**Additional Resources:**

Visions Volume 2, pp. 163-164

**Question:*****The Street Festival***

Frank is creating a game for people to play at a street festival in the community.

His idea is to have a bag with 9 baseballs, each with a number written on it. The number will represent the dollar amount which will be won by the participant who randomly draws it out of the bag.

The participant pays \$3.00 to play the game.

Frank numbered the balls as shown below:



Sue is running the festival and insists that the game be fair or in favour of the participant.

Frank says his game meets that condition. Sue disagrees.

Who is correct?

- Frank is correct.
- Sue is correct.

**General Strategies:**

1. Read the problem.
2. Highlight key words.
3. Identify the math topics.
4. Re-read the problem.
5. Define your steps (your game plan) – this is criteria 3.
6. Refer to your memory aid, as needed.
7. Solve.
8. If you get stuck on a calculation, pick a number and keep going.
9. Ask yourself whether your answer makes sense.
10. Write your answer statement.

Show any or all your work! Do not leave a blank page!

**My Strategies:**

**Answer and Solution:**

There are 5 possible outcomes:

- Picking a ball with 1 on it – which means –2 gain
- Picking a ball with 2 on it – which means –1 gain
- Picking a ball with 3 on it – which means 0 gain
- Picking a ball with 5 on it – which means +2 gain
- Picking a ball with 6 on it – which means +3 gain
- ... because it costs \$3.00 to play

The probability of each outcome is determined by how many of the 9 balls have that number on them.

- $P(1) = 2/9$  because there are 2 balls with a 1
- $P(2) = 2/9$  because there are 2 balls with a 2
- $P(3) = 3/9$  because there are 3 balls with a 3
- $P(5) = 1/9$  because there is 1 ball with a 5
- $P(6) = 1/9$  because there is 1 ball with a 6

$$P_1 \times (O_1) + P_2 \times (O_2) + \dots = \text{expected value}$$

$$P(1) \times (-2) + P(2) \times (-1) + P(3) \times (0) + P(5) \times (2) + P(6) \times (3) = EV$$

$$\frac{2}{9} \times (-2) + \frac{2}{9} \times (-1) + \frac{3}{9} \times (0) + \frac{1}{9} \times (2) + \frac{1}{9} \times (3) = EV$$

$$\frac{-4}{9} + \frac{-2}{9} + \frac{0}{9} + \frac{2}{9} + \frac{3}{9} = EV$$

$$\frac{-4}{9} + \frac{-2}{9} + \frac{0}{9} + \frac{2}{9} + \frac{3}{9} = EV$$

$$\frac{-1}{9} = EV$$

outcome	gain	probability	gain x probability
<b>Number on the ball</b>	$O_i$	$P_i$	$O_i \times P_i$
1	-\$2	2/9	-4/9
2	-\$1	2/9	-2/9
3	\$0	3/9	0/9
5	\$2	1/9	2/9
6	\$3	1/9	3/9
<b>total</b>		9/9	<b>-1/9</b>

Conclusion: since the expected value is negative, this game is not in the participant's favour.

- Frank is correct.
- Sue is correct.

**Suggested Strategies:**

Some of the key words:

- Game
- Random
- Fair
- Buying tickets
- Won

This is an expected value question.

What you need:

- All the possible outcomes
- The probability of each outcome

Things to check:

- Make sure all your probabilities add up to 1
- Make sure you have one term for each outcome (in this case 5)
- Make sure some of your terms are negative and some are positive
- Make sure you come to the correct conclusion; a "negative gain" is a "loss"
- You can also set up your solution in a table.

Note, that you can 'guess' this answer by checking one of the boxes, but you won't get credit for the guess – you have to show your reasoning in order to get marks.

**Additional Resources:**

Visions Volume 2, pp. 163-164

